Software Solutions to Problems on Heat Transfer

Radiation Heat Transfer – Part I

Dr. M. Thirumaleshwar

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RADIATION HEAT TRANSFER – PART-I

(Laws of Radiation and Radiation properties, View factors & Radiation energy exchange between black bodies)

Software Solutions to Problems on Heat Transfer: Radiation Heat Transfer – Part I
(Laws of Radiation and Radiation properties, View factors & Radiation energy exchange between black bodies)
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Contents

Preface to vol. 5

This is Vol. 5 of the book **"Software Solutions to Problems on Heat Transfer".**

In Vol. 1, problems on various aspects of CONDUCTION heat transfer were solved with Mathcad, EES, FEHT and EXCEL.

Vol. 2 contained solved problems on the topics of CONVECTION (i.e. Forced convection, Natural or Free convection).

In Vol. 3, we gave solved problems on the topics of BOILING and CONDENSATION. And,

In Vol. 4, we dealt with problems on the topic of HEAT EXCHANGERS.

Present Vol. viz. RADIATION HEAT TRANSFER contains problems solved on following topics:

    Vol. 5. RADIATION HEAT TRANSFER
    Radiation Heat Transfer – Equation summary
    5.A. Laws of Radiation & Radiation properties
    Problems solved with Mathcad (Prob. 5A.1 to 5A.10)
    Problems solved with EES (Prob. 5A.11 to 5A.12)
    Problems solved with EXCEL (Prob. 5A.13 to 5A.15)
    5.B. View Factors & Radiation energy exchange between Black bodies:
    Problems solved with Mathcad (Prob. 5B.1 to 5B.7)
    Problems solved with EES (Prob. 5B.8 to 5B.10)
    Problems solved with EXCEL (Prob. 5B.11 to 5B.18)
    5.C. Electrical network method & Radiation energy exchange between gray surfaces
    5.C.1 Radiation energy exchange in 2-zone enclosures
    5.C.2 Radiation energy exchange in 3-zone enclosures
    5.D. Radiation Shielding
    5.E. Radiation error in temperature measurement

    References

Here also, problems are solved using the popular software, viz. "Mathcad", "Engineering Equation Solver (EES)", and MS EXCEL spreadsheet. Comments are included generously in the codes so that the logic behind the solutions is clear. An introductory chapter in Part-I gives a brief overview of the software used.

Many Functions are written on View Factors for various geometries, Radiation exchange in 2_zone and 3-zone enclosures, Radiation shields etc. in Mathcad, EES and EXCEL-VBA. These Functions enable us to make the complicated calculations very easily.

As in Vol. I, II , III and IV, emphasis is given not only to solving a given problem but also to parametric analysis and graphical representation of results. Advantage of using Software to solve a variety of problems thus becomes evident. Also, problems with EXCEL spreadsheet are solved in greater details with relevant screen shots for immediate help to students, since EXCEL is available in practically every Personal Computer.

Acknowledgements: Firstly, I thank my students, since it is they who inspired me and motivated me.

Next, my thanks are due to the authorities at St. Joseph Engineering College, for their constant encouragement.

Also, my sincere thanks to **Bookboon.com** for publishing this book on the Internet. **Ms. Sophie** and her editorial team have been extremely helpful.

Finally, my heartfelt appreciation to **my wife, Kala,** but for whose unfailing and thoughtful support and encouragement, this work would not have been possible.

M. Thirumaleshwar
Author
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About the Author

Dr. M. Thirumaleshwar graduated in Mechanical Engineering from Karnataka Regional Engineering College, Surathkal, Karnataka, India, in the year 1965. He obtained M.Sc (cryogenics) from University of Southampton, U.K. and Ph.D.(cryogenics) from Indian Institute of Science, Bangalore, India.

He is a Fellow of Institution of Engineers (India), Life Member, Indian Society for Technical Education, and a Foundation Fellow of Indian Cryogenics Council.

He has worked in India and abroad on large projects in the areas involving heat transfer, fluid flow, vacuum system design, cryo-pumping etc.

He worked as Head of Cryogenics Dept. in Bhabha Atomic Research Centre (BARC), Bombay and Centre for Advanced Technology (CAT), Indore, from 1966 to 1992.

He worked as Guest Collaborator with Superconducting Super Collider Laboratory of Universities Research Association, in Dallas, USA from 1990 to 1993.

He also worked at the Institute of Cryogenics, Southampton, U.K. as a Visiting Research Fellow from 1993 to 1994.

He was Head of the Dept. of Mechanical Engineering, Fr. Conceicao Rodrigues Institute of Technology, Vashi, Navi Mumbai, India for eight years.

He also worked as Head of Dept. of Mechanical Engineering and Civil Engineering, and then as Principal, Vivekananda College of Engineering and Technology, Puttur (D.K.), India.

Presently, he is Professor and coordinator of Post-graduate program in the Dept. of Mechanical Engineering in St. Joseph Engineering College, Vamanjoor, Mangalore, India.

A book entitled **"Fundamentals of Heat and Mass Transfer"** authored by him and published by M/s Pearson Education, India (2006) **has been adopted as a Text book** for third year engineering students by the Visweswaraya Technological University (V.T.U.), Belgaum, India.

He has also written and published three book-lets entitled as follows:

1. Towards Excellence… How to Study (A Guide book to Students)
2. Towards Excellence… How to teach (A guide book to Teachers)
3. Towards Excellence… Seminars, GD's and Personal Interviews
   (A guide book to Professional and Management students)

Dr. M. Thirumaleshwar has attended several National and International conferences and has more than 50 publications to his credit.

About the Software used

Following three software are used while solving problems in this book:

1. Mathcad 2001 (Ref: www.ptc.com)
2. Engineering Equation Solver (EES) (Ref: www.fchart.com), and
3. Finite Element Heat Transfer (FEHT) (Ref: www.fchart.com)
4. MS EXCEL – (2007) Spreadsheet (Ref: Microsoft)

Trial versions of the first three software and detailed Instruction Manuals may be downloaded from the websites indicated. EXCEL is a very popular spreadsheet which comes bundled with MS OFFICE software, and is generally available in every computer.

See Part-I of this book for brief introduction to these four software.

While the information given there is enough to get going, for detailed instructions one should consult the respective Instruction manuals.

To the Student

Dear Student:

I would like to remind you that Heat Transfer is an important subject useful in many branches of engineering. It is also a subject in which you can score high marks in the examinations, since the question paper generally consists of derivations and numerical problems, almost in the ratio 50:50. Therefore, it requires that:

1) you are thorough with the derivations, and
2) skillful in solving numerical problems.

To be thorough with derivations, you should refer to well known, standard Text books on the subject of Heat Transfer (See References at the end of this book). **And, to develop your skill in solving problems… well, that is where I think that this book will help you.**

This book contains solutions to problems on heat transfer using four popular softwares, viz. Mathcad, Engineering Equation Solver (EES), Finite Element Heat Transfer (FEHT), and EXCEL spreadsheet. Trial versions of Mathcad, EES and FEHT can be downloaded from the websites indicated. EXCEL, which is a part of MS OFFICE, is generally preinstalled in most of the Personal Computers. Problems are chosen from the University question papers and standard heat transfer Text books.

Use of Software in solving problems has many advantages:

1. It helps in logical thinking
2. Problems are solved quickly and accurately
3. Parametric solutions (or 'what-if' solutions) are obtained easily
4. Solutions can be presented in tabular or graphical form, very easily and quickly
5. Once a particular type of problem is solved, solving a similar problem with different data input becomes very easy
6. Ease of getting solutions to problems in tabular or graphical form creates further interest and curiosity on the subject in the minds of students and encourages them to be creative and work further

How to use this Book?

You need not worry if you don't know about these softwares. Since each problem is solved systematically step by step, and is well commented, just reading through the solution will make the logic of the solution clear to you. That is the most important thing in solving the problems. ***Then, you must work out the problem yourself, by hand or using the software***. Of course, use of software has the above-mentioned advantages. *Simply reading the book won't do*. Have your favorite Text book nearby, in case you need to refer to it for any formulas or clarifications. There is no other 'easy method'.

As they say, *'there is no gain without pain!'*

Lastly, I would like to tell you how greatly I enjoyed solving the problems presented in this book using the softwares mentioned.

I hope that you too will enjoy as much as I did in solving these problems and get benefitted.

Good Luck!

Author

5   Radiation heat transfer

Learning objectives:

1. Radiation mode of heat transfer is radically different from conduction and convection modes in the sense that there is no need for a medium to be present for heat transfer to occur. But, just as conduction and convection heat transfers occur when there is a temperature gradient, net radiation heat transfer also occurs from a higher temperature level to a lower temperature level.

2. Radiation heat transfer is proportional to the fourth power of absolute temperature of the radiating surface. Therefore, radiation becomes the predominant mode of heat transfer when the temperature of the body is high.

3. Our interest in this chapter is on '**thermal radiation**', i.e. radiations in the wave length range of 0.1 to 100 microns.

4. Few important applications of radiation heat transfer:
   i)   industrial heating, such as in furnaces
   ii) industrial air-conditioning, where the effect of solar radiation has to be considered in calculating the heat loads
   iii) jet engine or gas turbine combustors
   iv) industrial drying
   v)   energy conversion with fossil fuel combustion etc.

5. In this Chapter, first, we solve problems on: fundamental laws governing radiation heat transfer, radiation properties of surfaces, such as absorptivity (α), emissivity (ε) and transmissivity (τ). Calculation of '**view factor**' for different geometries by analytical / graphical methods and by view factor algebra.

6. Next, we solve problems related to radiation heat transfer between surfaces in **two-surface and three-surface enclosures**, using the radiation network method. This method greatly simplifies the analysis and gives a 'physical feel' of the problem. Important practical examples of two-surface enclosure are: two infinite, parallel planes, long concentric cylinders and concentric spheres. Furnaces with re-radiating (insulated) surfaces are examples of three-surface enclosure.

7. '**Radiation shielding**' to reduce the radiation heat transfer between surfaces is studied next. Importance of radiation shielding in reducing the *radiation error in temperature measurement* is also studied.

8. Radiation has to be generally considered when the operating temperature level is high; as a rule, it will be prudent to check its relevance in problems involving natural convection and forced convection at high temperatures. Typical example is *heat transfer from walls and doors of furnaces*. In such problems, concept of '**radiation heat transfer coefficient**' simplifies the numerical calculations.

9. Problems on above topics are worked out using Mathcad, EES and EXCEL software. Several Functions/Procedures are written to make the complicated calculations very easy.

Formulas used: (Ref: 1)

Absorptivity, Reflectivity and Transmissivity:

$$\alpha + \rho + \tau = 1 \qquad\qquad\qquad \dots(13.1)$$

where, α = absorptivity = fraction of incident radiation absorbed

ρ = reflectivity = fraction of incident radiation reflected

τ = fraction of incident radiation transmitted

If $\tau = 1$, entire radiation passes through the body; such a body is **'transparent'** or **'diathermaneous'**.

If $\alpha = 1$, the body absorbs all the incident radiation and such a body is called a **'black body'.**

If $\rho = 1$, all the incident radiation is reflected, and it is a perfectly **'white body'**.

Emissive power (E): The 'total (or hemispherical) emissive power' is the total thermal energy radiated by a surface per unit time and per unit area, over all the wave lengths and in all directions.
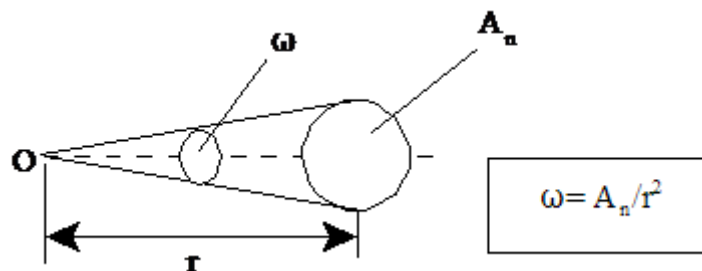
Solid angle: "Solid angle" is defined as a region of a sphere, which is enclosed by a conical surface with the vertex of the cone at the centre of the sphere. See Fig. below:

If a plane area A intercepts the line of propagation of radiation such that the normal to the surface makes an angle θ with the line of propagation, then we project the incident area normal to the line of propagation, such that, the solid angle is now defined as:

$$\omega = \frac{A \cdot \cos(\theta)}{r^2} \text{ sr.}$$
(13.3)

Note that, $A.\cos(\theta) = A_n$ is the projected area of the incident surface, normal to the line of propagation.

Intensity of radiation (I_b): Intensity of radiation for a black body, I_b is defined as the energy radiated per unit time per unit solid angle per unit area of the emitting surface projected normal to the line of view of the receiver from the radiating surface.

Mathematically, this is expressed as:

$$I_b = \frac{dQ_b}{(dA \cdot \cos(\theta)) \cdot d\omega} \text{ W/(m2.sr)}$$
(13.4)

i.e.
$$I_b = \frac{dE_b}{\cos(\theta) \cdot d\omega} \text{ W/(m2.sr)}$$
(13.5)

Note that Emissive power E_b of a black body refers to unit *surface area* whereas Intensity I_b of a black surface refers to unit *projected area*.

$I_b\lambda$ is the intensity of blackbody radiation for radiation of a given wave length λ. And, I_b is the summation over all the wave lengths, i.e.

$$I_b = \int_0^\infty I_{b\lambda}\, d\lambda \text{ W/(m2.sr)}$$
(13.6)

Lambert's cosine law: "A diffuse surface radiates energy such that the rate of energy radiated in a direction θ from the normal to the surface is proportional to the cosine of the angle θ". i.e.

$$Q_\theta = Q_n \cdot \cos(\theta)$$
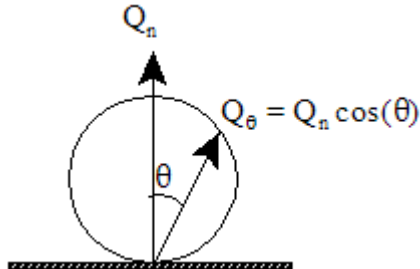
13.3   Laws of black body radiation:

Planck's Law for spectral distribution:

Radiation energy emitted by a black surface depends on the wave length, temperature of the surface and the surface characteristics.

Planck's distribution Law relates to the **spectral black body emissive power, $E_b\lambda$** defined as 'the amount of radiation energy emitted by a black body at an absolute temperature T per unit time, per unit surface area, per unit wave length about the wave length λ'. Units of $E_b\lambda$ are: W/(m².μm). The first subscript 'b' indicates black body and the second subscript 'λ' stands for given wavelength, or monochromatic. Planck derived his equation for $E_b\lambda$ in 1901 in conjunction with his 'quantum theory'.
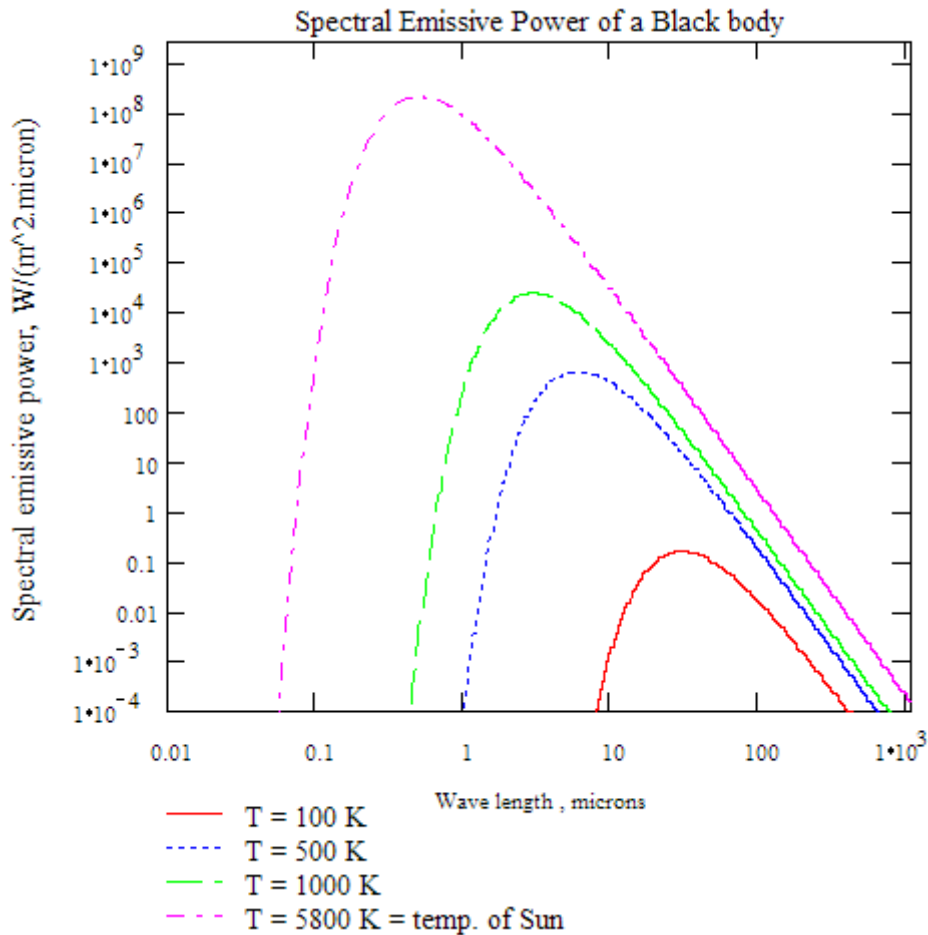
Planck's distribution law is expressed as:

$$E_{b\lambda}(\lambda) = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1} \qquad \text{W/(m2.μm).....(13.7)}$$

where, $C_1 = 3.742 \cdot 10^8$ W.μm4/m2

and, $C_2 = 1.4387 \cdot 10^4$ μm.K

Plots of $E_b\lambda$ vs. λ for a few different temperatures are shown in Fig. below:

Spectral Emissive Power of a Black body

Wein's displacement law:

It is clear from Fig. 13.7 that the spectral distribution of emissive power of a black body at a given absolute temperature goes through a maximum. To find out the value of λ_{max}, the wave length at which this maximum occurs, differentiate Planck's equation w.r.t. λ and equate to zero. Finally, we get:

$$\lambda_{max} \cdot T = 2898 \quad \mu m.K. \tag{13.11}$$

Wein's displacement law is stated as: "product of absolute temperature and wave length at which emissive power of a black body is a maximum, is constant".

Value of maximum monochromatic emissive power of a black body at a given temperature is obtained by substituting this value of $\lambda_{max}T$ (= 2898 $\mu m.K$) in Planck's equation. And, we get:

$$E_{b\lambda_{max}} = 1.287 \cdot 10^{-5} \cdot T^5 \quad W/m^3 \tag{13,12}$$

Stefan-Boltzmann Law:

$$E_b = \sigma \cdot T^4 \quad W/m^2 \tag{13.13}$$

$$\text{where,} \quad \sigma = 5.67 \cdot 10^{-8} \quad W/(m^2.K^4)$$

σ is known as 'Stefan-Boltzmann constant'.

Net radiant energy exchange between two black bodies at temperatures T_1 and T_2 is given by:

$$Q_{net} = \sigma \cdot \left(T_1^4 - T_2^4 \right) \ W/m^2 \tag{13.14}$$

Radiation from a wave band:

Often, it is required to know the amount of radiation emitted in a given wave band, i.e. in a wave length interval between λ_1 and λ_2. This is expressed as a fraction of the total emissive power and is written as $F\lambda_{1\text{-}}\lambda_2$. Then, we can write:

$$F_{\lambda 1\_\lambda 2} = \frac{\displaystyle\int_{\lambda 1}^{\lambda 2} E_{b\lambda}\, d\lambda}{\displaystyle\int_{0}^{\infty} E_{b\lambda}\, d\lambda} = \frac{1}{\sigma \cdot T^4} \cdot \int_{\lambda 1}^{\lambda 2} E_{b\lambda}\, d\lambda$$

i.e.

$$F_{\lambda 1\_\lambda 2} = \frac{1}{\sigma \cdot T^4} \cdot \left(\int_{0}^{\lambda 2} E_{b\lambda}\, d\lambda - \int_{0}^{\lambda 1} E_{b\lambda}\, d\lambda \right)$$

i.e.

$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2} - F_{0\_\lambda 1} \tag{13.5}$$

Above formula is not very convenient to use, since $E_b\lambda$ depends on absolute temperature T, and it is not practicable to tabulate $F_{0\text{-}\lambda}$ for each T. This difficulty is overcome by expressing $F_{0\text{-}\lambda}$ as follows:

$$F_{0\_\lambda} = \frac{\displaystyle\int_{0}^{\lambda} E_{b\lambda}\, d\lambda}{\sigma \cdot T^4} = \frac{\displaystyle\int_{0}^{\lambda \cdot T} E_{b\lambda}\, d(\lambda \cdot T)}{\sigma \cdot T^5}$$

i.e.

$$F_{0\_\lambda} = f(\lambda \cdot T) \tag{13.16}$$

i.e. Now, $F_{0\text{-}}\lambda$ is expressed as a function of the product of wave length and absolute temperature ($= \lambda.T$) only.

Values of $F_{0-\lambda}$ vs. $\lambda.T$ are tabulated in Table 13.2 below:

Radiation Functions			
λT (μm.K)	$F_{0-\lambda}$	λT (μm.K)	$F_{0-\lambda}$
400	0	7000	0.8081
600	0	7200	0.8192
800	0.000016	7400	0.8295
1000	0.00032	7600	0.848
1200	0.00213	7800	0.848
1400	0.0078	8000	0.8563
1600	0.0197	8500	0.8746
1800	0.0393	9000	0.89
2000	0.0667	9500	0.9031
2200	0.1009	10000	0.9142
2400	0.1403	10500	0.9237
2600	0.1831	11000	0.9319
2800	0.2279	11500	0.9399
3000	0.2732	12000	0.9451
3200	0.3181	12500	0.9505
3400	0.3617	13000	0.9551
3600	0.4036	13500	0.9592
3800	0.4434	14000	0.9628
4000	0.4809	14500	0.9661
4200	0.516	15000	0.9689
4400	0.5488	16000	0.9738
4600	0.5793	17000	0.9776
4800	0.6075	18000	0.9808
5000	0.6337	19000	0.9834
5200	0.659	20000	0.9855
5400	0.6804	25000	0.9922
5600	0.701	30000	0.9953
5800	0.7201	35000	0.9969
6000	0.7378	40000	0.9979
6200	0.7541	45000	0.9985
6400	0.7962	50000	0.9989
6600	0.7832	75000	0.9997
6800	0.7961	100000	0.9999

Table 13.2.

Note in the above Table that the units of product $\lambda.T$ is (micron.Kelvin).

Therefore,

$$F_{\lambda 1\_\lambda 2} = F_{\lambda 1.T\_\lambda 2.T} = \frac{1}{\sigma} \cdot \left[\int_0^{\lambda 2 \cdot T} \frac{E_{b\lambda}}{T^5} d(\lambda \cdot T) - \int_0^{\lambda 1 \cdot T} \frac{E_{b\lambda}}{T^5} d(\lambda \cdot T) \right]$$

i.e.

$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2.T} - F_{0\_\lambda 1.T} \tag{13.17}$$

Relation between Radiation intensity and Emissive power:

$$E_b = \pi \cdot I_b \tag{13.21}$$

i.e. Total emissive power of a black (diffuse) surface is equal to π times the intensity of radiation.

This is an important relation, which will be used while calculating the view factors required to determine net energy exchange between surfaces.

Total hemispherical emissivity (ε) of a surface is the average emissivity over all directions and all wave lengths and is expressed as:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma \cdot T^4} \tag{13.22}$$

where E(T) is the emissive power of the real surface.

Similarly, spectral emissivity is defined as:

$$\varepsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)}$$

(13.23)

Emissivity values for a few surfaces at room temperature are given in handbooks.

A surface is said to be **gray** if its properties are independent of wave length, and a surface is **diffuse** if its properties are independent of direction.

To calculate the average emissivity:

Let the variation of spectral emissivity with wave length be as follows:

ε_1 = constant, $0 \leq \lambda \leq \lambda_1$

ε_2 = constant, $\lambda_1 \leq \lambda \leq \lambda_2$

ε_3 = constant, $\lambda_2 \leq \lambda \leq \infty$

Then, the average emissivity is calculated using eqn (13.24) as follows:

$$\varepsilon(T) = \frac{\varepsilon_1 \cdot \int_0^{\lambda_1} E_{b\lambda}(T)d\lambda}{\sigma \cdot T^4} + \frac{\varepsilon_2 \cdot \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(T)d\lambda}{\sigma \cdot T^4} + \frac{\varepsilon_3 \cdot \int_{\lambda_2}^{\infty} E_{b\lambda}(T)d\lambda}{\sigma \cdot T^4}$$

(13.25)

i.e. $\varepsilon(T) = \varepsilon_1 \cdot F_{0-\lambda_1}(T) + \varepsilon_2 \cdot F\lambda_1{}_{-\lambda_2}(T) + \varepsilon_3 \cdot F\lambda_2{}_{-\infty}(T)$

(13.26)

Factors $F_{0-\lambda_1}(T)$ etc. can easily be determined using Tables in handbooks.

13.3.7 Kirchoff's Law:

$$\varepsilon(T) = \alpha(T)\ldots..(13.27)$$

Kirchoff's Law states that the total hemispherical emissivity, ε of a gray surface at a temperature T is equal to its absorptivity, α for black body radiation from a source at the same temperature T.

The View factor:

View factor is also known by other names such as: 'configuration factor', 'shape factor', 'angle factor' etc.

View factor is defined as *the fraction of radiant energy leaving one surface which strikes a second surface directly*.

Here, 'directly' means that reflection or re-radiated energy is not considered. View factor is denoted by F_{12}, where the first subscript, 1 stands for the emitting surface, and the second subscript, 2 stands for the receiving surface.

View factor between Infinitesimal areas:

(Such a situation is encountered even when finite areas are involved, when the distance between these two areas 'r', is very large.)

$$F_{dA1\_dA2} = \frac{\cos(\phi_1) \cdot \cos(\phi_2) \cdot dA_2}{\pi \cdot r^2} \qquad ....(13.31)$$

Note that the View factor involves geometrical quantities only.

View factor between Infinitesimal and finite area:

$$F_{dA1-A2} = \int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)\,dA_2}{\pi\,r^2} \ldots\ldots(13.32)$$

Practical situation of calculating view factors between infinitesimal to finite areas are encountered the case of a small thermocouple bead located inside a pipe or a small, spherical point source radiator located by the side of a wall etc.

View factor from Finite to finite area:

$$F_{A1-A2} = \frac{1}{\pi\,A_1} \int_{A1}\int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)}{r^2}\,dA_1 dA_2 \cdots\cdots(13.33)$$

In general, we write eqn. (13.33) compactly as:

$$F_{12} = \frac{1}{\pi\,A_1} \int_{A1}\int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)}{r^2}\,dA_1 dA_2 \cdots\cdots(13.34)$$

Reciprocity theorem:

$$A_1 \cdot F_{12} = A_2 \cdot F_{21} \qquad .....(13.36)$$

This is a very useful and important relation. It helps one to find out one of the view factors when the other one is known. In practice, one of the view factors which is easier to calculate is obtained first, and the other view factor is found out next, by using the reciprocity theorem.

Note: It is easier to remember the view factor relation given in eqn. (13.34) as:

$$A_{11}F_{12} = \int_{A1}\int_{A2} \frac{\cos(\phi_1)\cos(\phi_2)}{\pi\,r^2}\,dA_1 dA_2 \cdots\cdots(13.37)$$

Radiation energy exchange between black bodies:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot T_1^4 - A_2 \cdot F_{21} \cdot \sigma \cdot T_2^4$$

But, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ by reciprocity theorem

Therefore,

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4 \right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4 \right) \qquad W....(13.40)$$

Properties of View factor and view factor algebra:

Properties of view factor:

1) The view factor depends only on the geometrics of bodies involved and not on their temperatures or surface properties.
2) Between two surfaces that exchange energy by radiation, the mutual shape factors are governed by the **'reciprocity relation'**, viz. $A_1.F_{12} = A_2.F_{21}$.

3) **When a convex surface 1 is completely enclosed by another surface 2**, it is clear from Fig. 13.14(a) that all of the radiant energy emitted by surface 1 is intercepted by the enclosing surface 2. Therefore, **view factor of surface 1 w.r.t. surface 2 is equal to unity. i.e. $F_{12} = 1$.** And, the view factor of surface 2 w.r.t. surface 1 is then easily calculated by applying the reciprocity relation, i.e. $A_1.1 = A_2.F_{21}$, or, $F_{21} = A_1/A_2$.

4) Radiation emitted from a flat surface never falls directly on that surface (see Fig. 13. 14 (b)), **i.e. View factor of a flat surface w.r.t. itself is equal to zero, i.e. $F_{11} = 0$.** This is valid for a convex surface too, as shown in Fig. 13.14(c).

5) **For a concave surface,** it is clear from Fig. 13.14(d) that F_{11} is not equal to zero since some fraction of radiation emitted by a concave surface does fall on that surface directly.
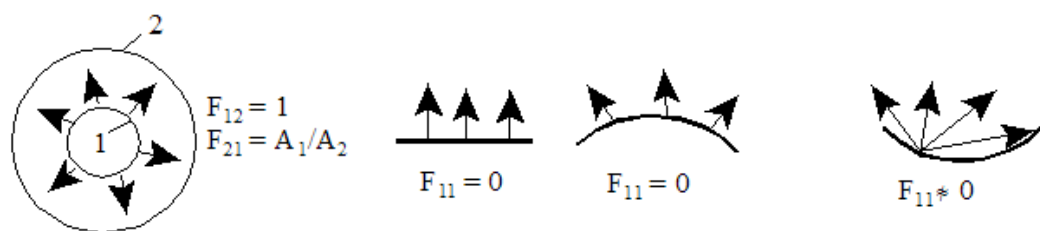
(a) Surface 1 completely enclosed by surface 2

(b) Flat surface (c) Convex surface (d) Concave surface

Fig. 13.14 View factors for a few surfaces

6) **If two, plane surfaces A1 and A2 are parallel to each other and separated by a short distance between them,** practically all the radiation issuing from surface 1 falls directly on surface 2, and vice –versa. Therefore, **$F_{12} = F_{21} = 1$**
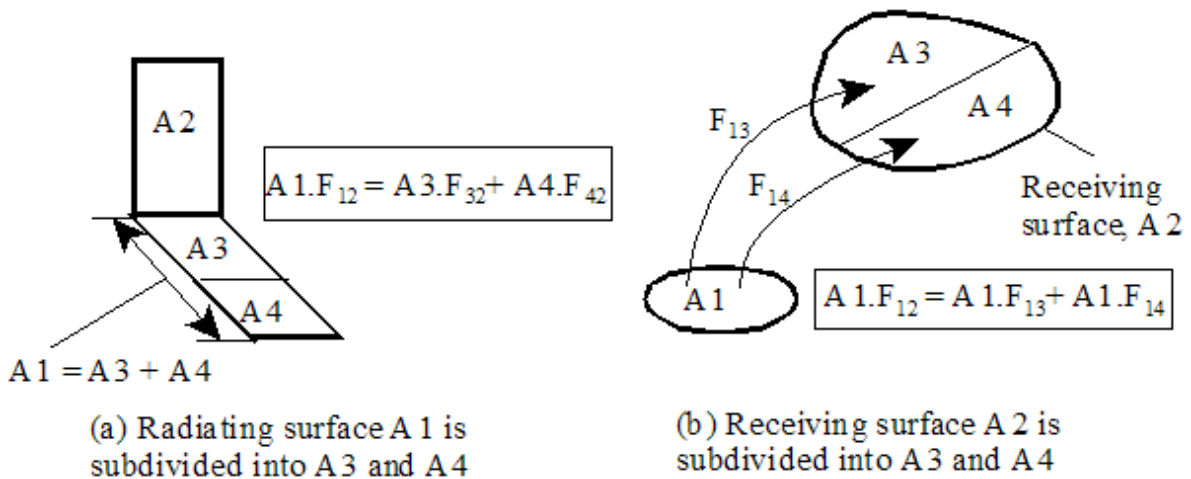

(a) Radiating surface A 1 is subdivided into A 3 and A 4

(b) Receiving surface A 2 is subdivided into A 3 and A 4

Fig. 13.15 View factors for subdivided surfaces

7) **When the radiating surface 1 is divided into, say, two sub-areas A3 and A4,** as shown in Fig. 13.15(a), we have:

$$A_1 \cdot F_{12} = A_3 \cdot F_{32} + A_4 \cdot F_{42} \quad \ldots \qquad (13.41)$$

Obviously, $F_{12} \neq F_{32} + F_{42}$.

8) **Instead, if the receiving surface A2 is sub-divided into parts A3 and A4** as shown in Fig. 13.15 (b), we have:

$$A_1 \cdot F_{12} = A_1 \cdot F_{13} + A_1 \cdot F_{14}$$

i.e. $\qquad\qquad\qquad\qquad \ldots \qquad (13.42)$

$$F_{12} = F_{13} + F_{14}$$

i.e. view factor from the emitting surface 1 to a sub-divided receiving surface is simply equal to the sum of the individual shape factors from the surface 1 to the respective parts of the receiving surface. This is known as **'Superposition rule'**.

9) **Symmetry rule:** If two (or more) surfaces are symmetrically located w.r.t. the radiating surface 1, then the view factors from surface 1 to these symmetrically located surfaces are identical. A close inspection of the geometry will reveal if there is any symmetry in a given problem.

10) **Summation rule:** Since radiation energy is emitted from a surface in all directions, invariably, we consider the emitting surface to be part of an enclosure. Even if there is an opening, we consider the opening as a surface with the radiative properties of that opening. Then, the conservation of energy principle requires that sum of all the view factors from the surface 1 to all other surfaces forming the enclosure, must be equal to 1. See Fig. 13.16, where the interior surface of a completely enclosed space is sub-divided into n parts, each of area A1, A2, A3…An.
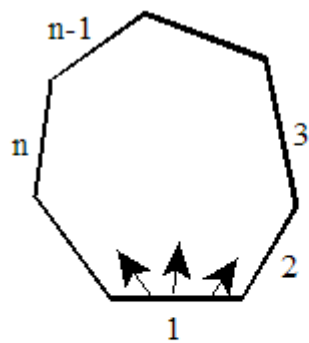
$$\sum_{j=1}^{n} F_{ij} = 1 \quad i = 1, 2, 3, \ldots . n$$

Fig. 13.16 View factor – summation rule for radiation in an enclosure

Then,

$$F_{11} + F_{12} + \ldots + F_{1n} = 1$$

$$F_{21} + F_{22} + \ldots + F_{2n} = 1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

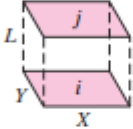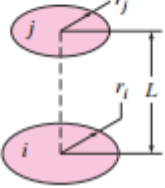$$F_{n1} + F_{n2} + \ldots + F_{nn} = 1 \ldots \tag{13.43}$$

i.e.

$$\sum_{j=1}^{n} F_{ij} = 1 \quad i = 1, 2, 3, \ldots . n \ldots\ldots (13.44)$$
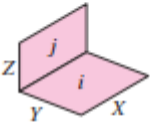
11) In an enclosure of 'n' black surfaces, maintained at temperatures T_1, T_2,....T_n, *net* radiation from any surface, say surface 1, is given by summing up the net radiation heat transfers from surface 1 to each of the other surfaces of the enclosure:

$$Q_{1_{net}} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) + A_1 \cdot F_{13} \cdot \sigma \cdot \left(T_1^4 - T_3^4\right) + A_1 \cdot F_{14} \cdot \sigma \cdot \left(T_1^4 - T_4^4\right) \dots$$
$$+ A_1 \cdot F_{1n} \cdot \sigma \cdot \left(T_1^4 - T_n^4\right)$$

....(13.45)

Note: Often, while solving radiation problems, determination of the view factor is the most difficult part. It will be useful to keep in mind the definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule while attempting to find out the view factors.
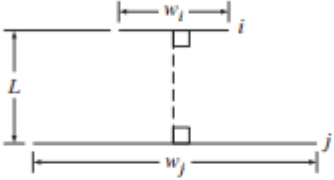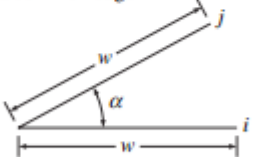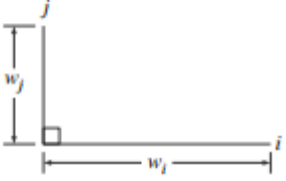
View factor relations for a few geometries are given below: (Ref: Cengel)
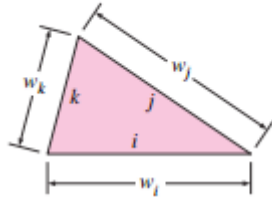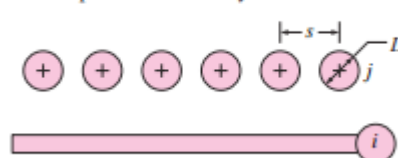
TABLE 22–1

View factor expressions for some common geometries of finite size (3D)

Geometry	Relation
Aligned parallel rectangles	$\overline{X} = X/L$ and $\overline{Y} = Y/L$ $$F_{i \to j} = \frac{2}{\pi \overline{X} \overline{Y}} \left\{ \ln \left[\frac{(1 + \overline{X}^2)(1 + \overline{Y}^2)}{1 + \overline{X}^2 + \overline{Y}^2} \right]^{1/2} \right.$$ $$+ \overline{X}(1 + \overline{Y}^2)^{1/2} \tan^{-1} \frac{\overline{X}}{(1 + \overline{Y}^2)^{1/2}}$$ $$+ \overline{Y}(1 + \overline{X}^2)^{1/2} \tan^{-1} \frac{\overline{Y}}{(1 + \overline{X}^2)^{1/2}}$$ $$\left. - \overline{X} \tan^{-1} \overline{X} - \overline{Y} \tan^{-1} \overline{Y} \right\}$$
Coaxial parallel disks	$R_i = r_i/L$ and $R_j = r_j/L$ $$S = 1 + \frac{1 + R_j^2}{R_i^2}$$ $$F_{i \to j} = \frac{1}{2} \left\{ S - \left[S^2 - 4 \left(\frac{r_j}{r_i} \right)^2 \right]^{1/2} \right\}$$

Perpendicular rectangles with a common edge	$H = Z/X$ and $W = Y/X$
	$F_{i \to j} = \frac{1}{\pi W} \left(W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} \right.$ $- (H^2 + W^2)^{1/2} \tan^{-1} \frac{1}{(H^2 + W^2)^{1/2}}$ $+ \frac{1}{4} \ln \left\{ \frac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \right.$ $\times \left[\frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{W^2}$ $\left. \left. \times \left[\frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{H^2} \right\} \right)$

TABLE 22–2

View factor expressions for some infinitely long (2D) geometries

Geometry	Relation
Parallel plates with midlines connected by perpendicular line	$W_i = w_i/L$ and $W_j = w_j/L$ $F_{i \to j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined plates of equal width and with a common edge	$F_{i \to j} = 1 - \sin \frac{1}{2}\alpha$
Perpendicular plates with a common edge	$F_{i \to j} = \frac{1}{2} \left\{ 1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i} \right)^2 \right]^{1/2} \right\}$

Three-sided enclosure	$F_{i \rightarrow j} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders	$F_{i \rightarrow j} = 1 - \left[1 - \left(\dfrac{D}{s} \right)^2 \right]^{1/2}$ $+ \dfrac{D}{s} \tan^{-1} \left(\dfrac{s^2 - D^2}{D^2} \right)^{1/2}$

View factor for two aligned parallel, rectangles of equal size:
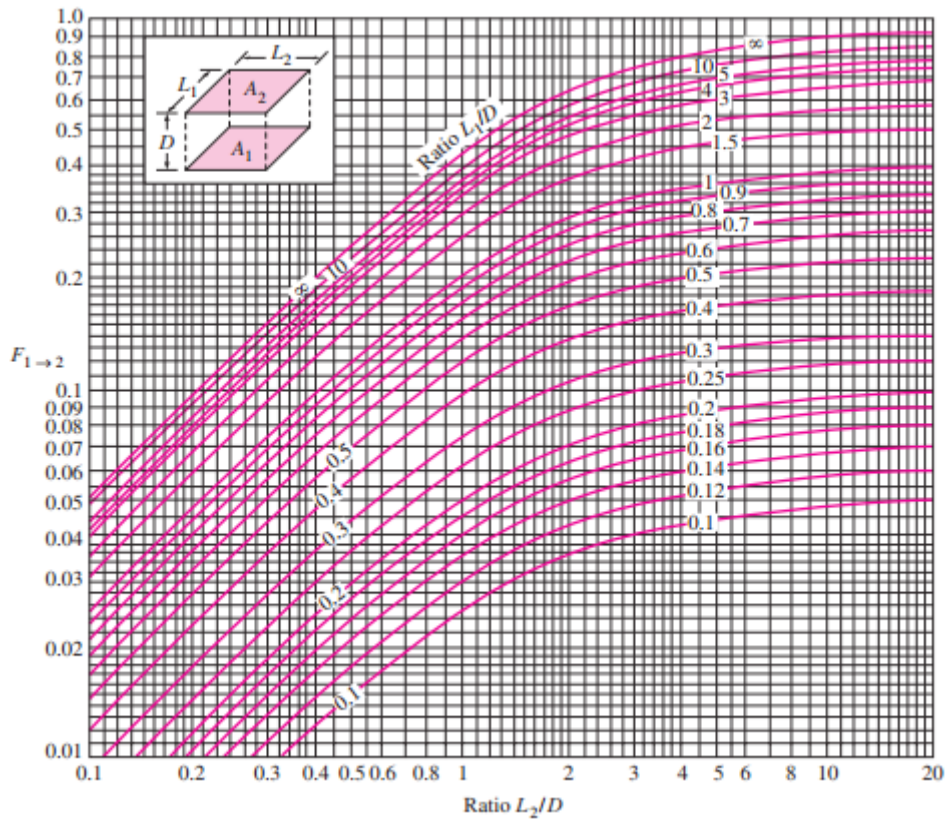
View factor between two coaxial parallel discs:
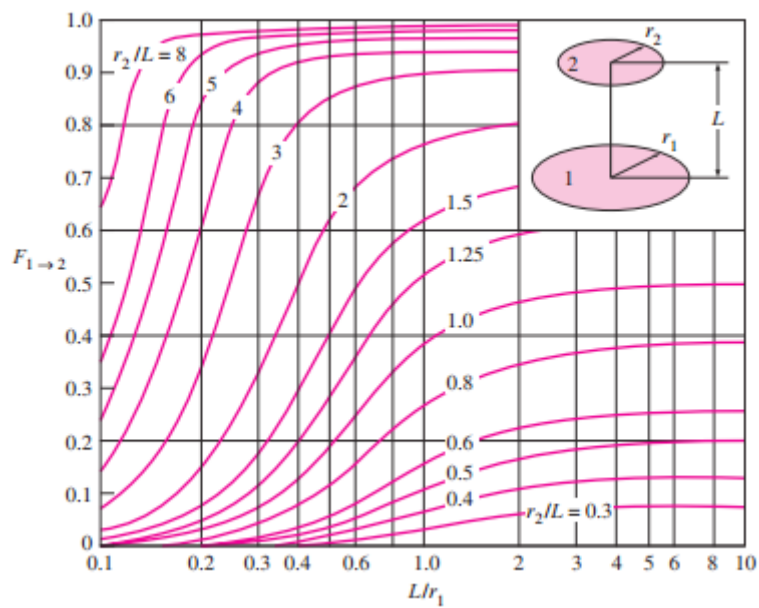
View factor between two perpendicular rectangles with a common edge:
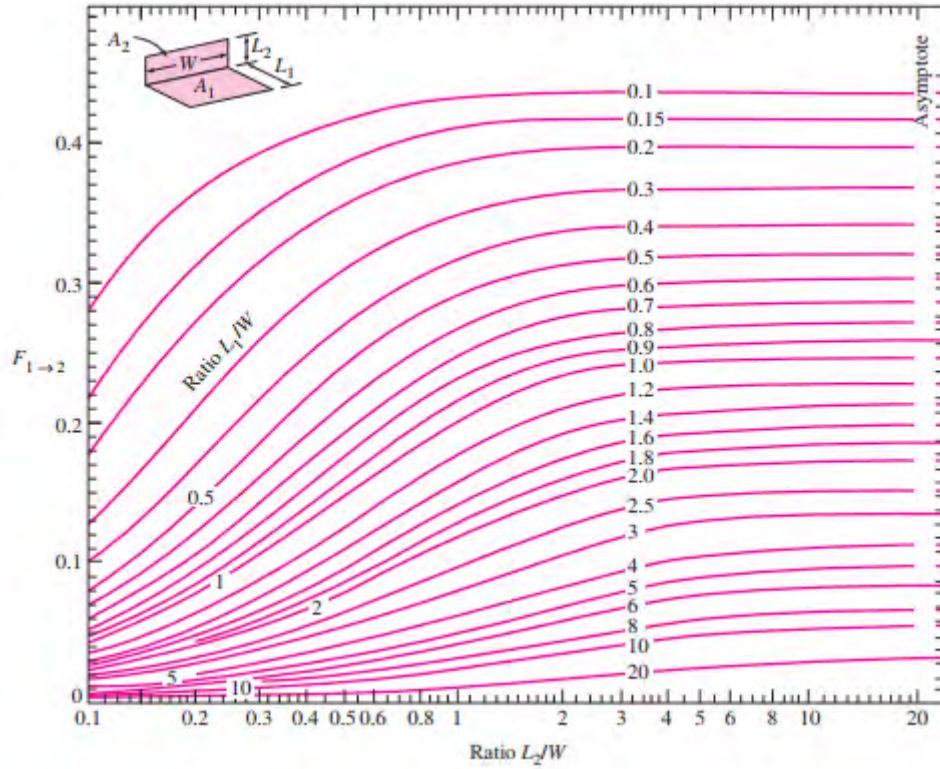
View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder (b) outer cylinder to itself:
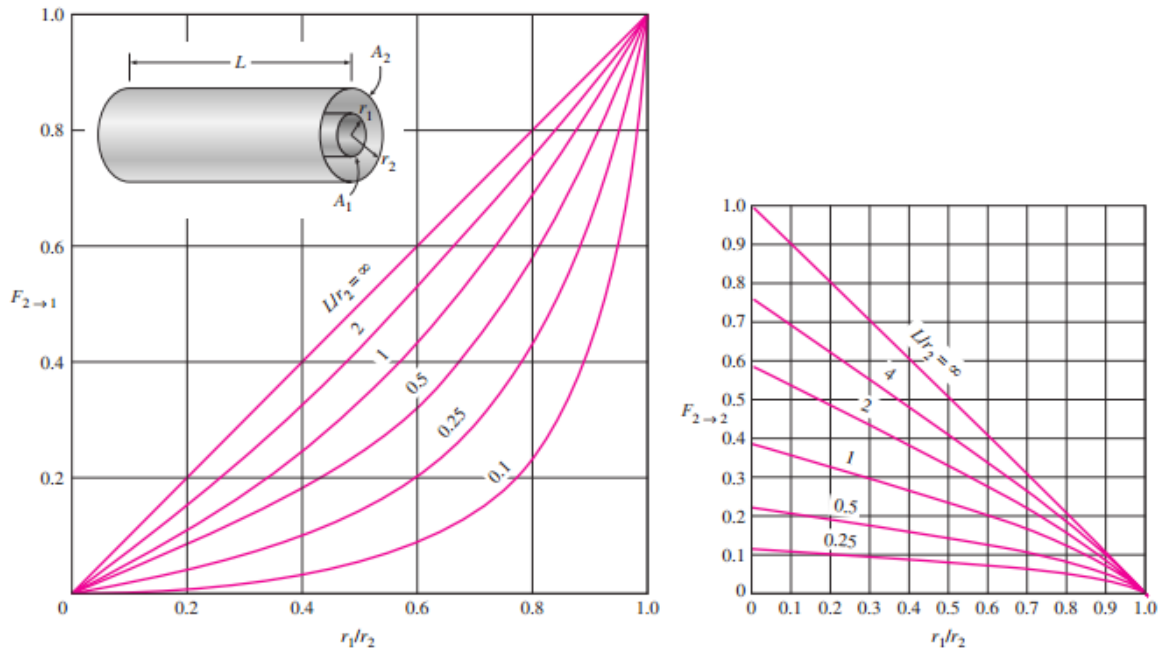
Radiation heat exchange between gray surfaces: (Ref: 1, 3)

Radiation exchange between small, gray surfaces:

$$Q_{12} = \varepsilon_1 \cdot \varepsilon_2 \cdot A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = \varepsilon_1 \cdot \varepsilon_2 \cdot A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \qquad ....(13.47)$$

Remember that:

$$A_1 \cdot F_{12} = A_2 \cdot F_{21} \qquad ...\text{by reciprocity}$$

The product, $(\varepsilon_1 . \varepsilon_2)$ is known as 'equivalent emissivity (ε_{eq})' for a system of two 'small' gray bodies.

The electrical network method:

Definitions:

1. **Irradiation, (G):** is the total radiation incident upon a surface per unit time, per unit area (W/m^2).
2. **Radiosity, (J):** is the total radiation leaving a surface, *with no regard for its origin* (i.e., reflected *plus* emitted from the surface) per unit time, per unit area (W/m^2).

Fig. 13.25 Irradiation and Radiosity

3. **Surface resistance:**

$$Q = \frac{E_b - J}{\frac{(1-\varepsilon)}{A \cdot \varepsilon}} \qquad W \ldots (13.48)$$

By analogy with Ohm's law:

$$R = \frac{(1-\varepsilon)}{A \cdot \varepsilon} \qquad \ldots \text{surface resistance}$$
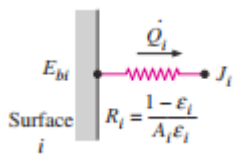
$J_i = E_{bi} \qquad \ldots$ for a black body....(13.49)

$J_i = E_{bi} = \sigma \cdot T_i^4 \qquad \ldots$ for a re-radiating surface....(13.50)

4. **Space resistance:**
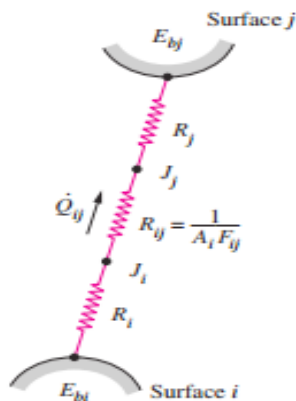
$$Q_{ij} = \frac{J_i - J_j}{\dfrac{1}{(A_i \cdot F_{ij})}} \quad W....(13.52)$$

By analogy with Ohm's law:

$$Q_{ij} = \frac{J_i - J_j}{R_{ij}} \quad W$$

$$\text{where,} \quad R_{ij} = \frac{1}{A_i \cdot F_{ij}} \quad ...(13.53)$$

R_{ij} is known as **'space resistance'** and it represents the resistance to radiative heat flow between the radiosity potentials of the two surfaces, due to their relative orientation and spacing.

For a N surface enclosure, net heat transfer from surface i should be equal to the sum of net heat transfers from that surface to the remaining surfaces. i.e.

$$Q_i = \sum_{j=1}^{N} Q_{ij} = \sum_{j=1}^{N} A_i \cdot F_{ij} (J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \quad ...W....(13.54)$$

$$\text{i.e.} \quad \frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \quad W...(13.55)$$

where R_i is the surface resistance and R_{ij} is the space resistance.

This situation is shown in Fig. (13.27).
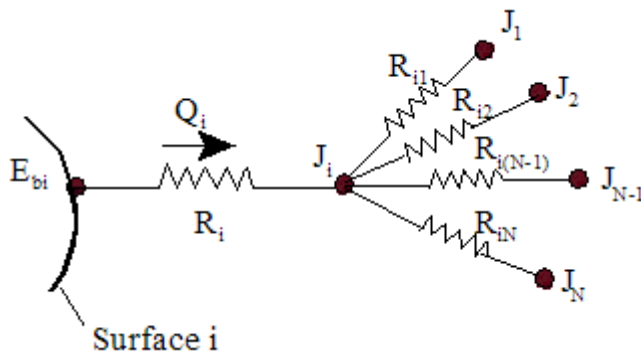
Fig. 13.27 Radiation heat transfer from surface i to other surfaces in a N-surface enclosure

Radiation heat exchange in two-zone enclosures: (Ref: 1, 3)
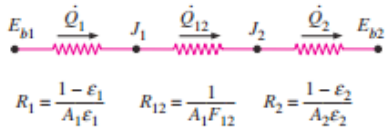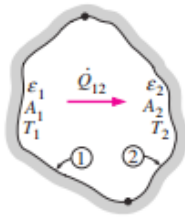
$$Q_{12} = Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

i.e.
$$Q_{12} = \frac{E_{b1} - E_{b2}}{\dfrac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1} + \dfrac{1}{A_1 \cdot F_{12}} + \dfrac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

i.e.
$$Q_{12} = \frac{\sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1} + \dfrac{1}{A_1 \cdot F_{12}} + \dfrac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}} \qquad W \ldots\ldots (13.56)$$

A few special cases of two-surface enclosure:

Here, basic, governing eqn. is eqn.(13.56), which is modified depending upon the case considered.

Case (i): Radiant heat exchange between two black surfaces:

For a black body, $\varepsilon = 1$, and $J = E_b$, as explained earlier. i.e. surface resistance

$[= (1 - \varepsilon)/(A.\varepsilon)]$ of a black body is zero.

Then, from eqn. (13.56), we get:

$$Q_{12} = \frac{\sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{A_1 \cdot F_{12}}}$$

i.e. $\quad Q_{12} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad$ W....for two black surfaces forming an enclosure....(13.57)

Next, we shall consider four cases of practical interest where the view factor between the inner surface 1 and the outer surface 2 (i.e. F_{12}) is equal to 1, and also the net radiation from a gray cavity:

Case (ii): Radiant heat exchange for a small object in a large cavity:

A practical example of a small object in a large cavity is the case of a steam pipe passing through a large plant room.

For this case, we have:

$$\frac{A_1}{A_2} = 0$$

and, $\quad F_{12} = 1$

And, eqn.(13.56) becomes:

$$Q_{12} = A_1 \cdot \sigma \cdot \varepsilon_1 \cdot \left(T_1^4 - T_2^4\right) \quad \text{....for small object in a large cavity.....(13.58)}$$

Case (iii): Radiant heat exchange between infinitely large parallel plates:

In this case, $A_1 = A_2 = A$, say, and $F_{12} = 1$

Then, eqn. (13.56) becomes:

$$Q_{12} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \quad \text{....for infinitely large parallel plates.....(13.59)}$$

Case (iv): Radiant heat exchange between infinitely long concentric cylinders:

In this case:

$$F_{12} = 1$$

Then, eqn. (13.56) becomes:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \left(\dfrac{A_1}{A_2}\right) \cdot \left(\dfrac{1}{\varepsilon_2} - 1\right)} \qquad \text{....for infinitely long concentric cylinders.....(13.60)}$$

where

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Remember that A_1 refers to the inner (or enclosed) surface.

Case (v): Radiant heat exchange between concentric spheres:

In this case:

$$F_{12} = 1$$

Then, eqn. (13.56) becomes

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \left(\dfrac{A_1}{A_2}\right) \cdot \left(\dfrac{1}{\varepsilon_2} - 1\right)} \qquad \text{....for concentric spheres.....(13.61)}$$

where

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Remember, again, that A_1 refers to the inner (or enclosed) surface.

Case (vi): Energy radiated from a gray cavity:
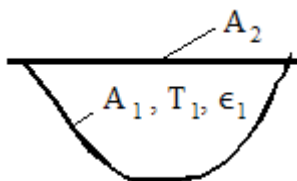
(a) Gray cavity
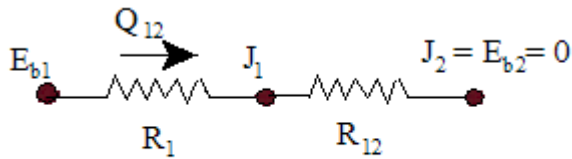
(b) Radiation network

$$R_1 = (1 - \epsilon_1)/(A_1 . \epsilon_1)$$

$$R_{12} = 1/(A_1 . F_{12})$$

$$Q_{12} = A_1 \cdot \epsilon_1 \cdot \sigma \cdot T_1^4 \cdot \left[\frac{1 - F_{11}}{1 - (1 - \epsilon_1) \cdot F_{11}} \right] \quad W \ldots\text{net radiation from gray cavity} \ldots (13.62)$$

Eqn. (13.62) is an *important result,* which gives net radiation from a gray cavity to surrounding space. If it is desired, for example, to calculate the net radiation from a blind hole drilled in a flange, then the relation to use is the eqn. (13.62).

Radiation heat exchange in three zone enclosures: [Ref: 1]

Fig.(a)

$R_1 = (1 - \epsilon_1)/(A_1 . \epsilon_1)$

$R_2 = (1 - \epsilon_2)/(A_2 . \epsilon_2)$

$R_3 = (1 - \epsilon_3)/(A_3 . \epsilon_3)$

$R_{12} = 1/(A_1 . F_{12})$

$R_{13} = 1/(A_1 . F_{13})$

$R_{23} = 1/(A_2 . F_{23})$

Fig.(b)
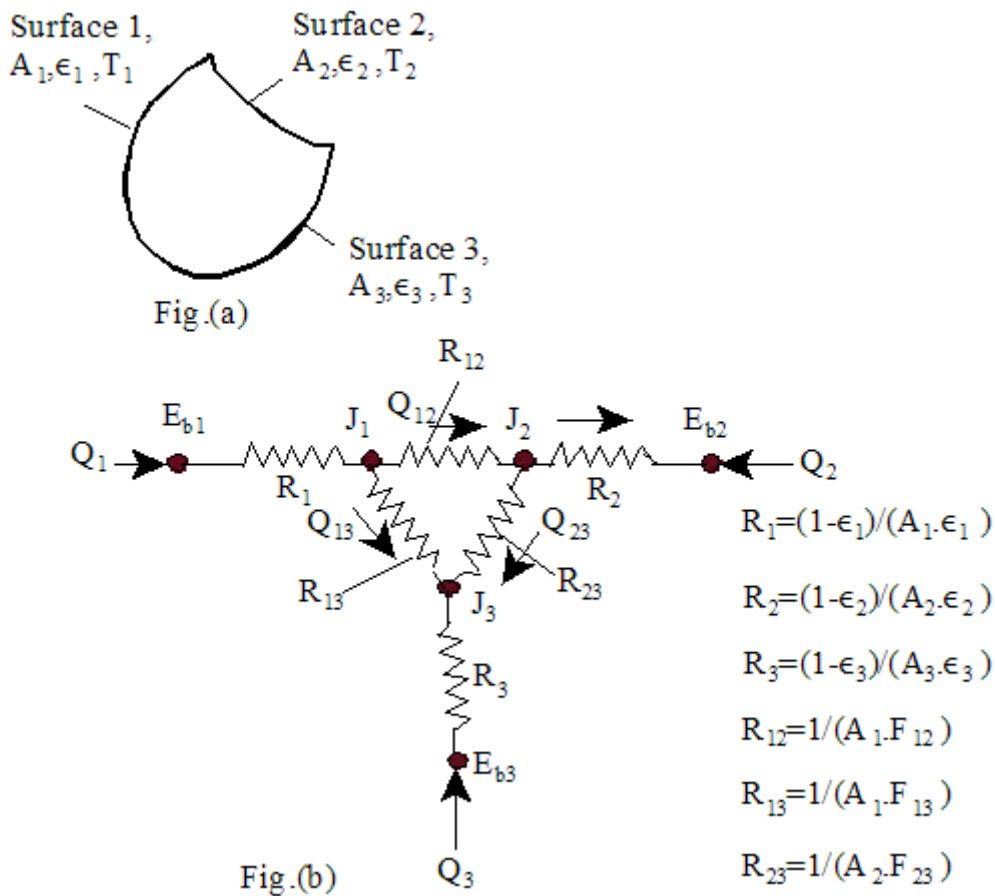
Temperature of each surface is known; i.e. emissive power E_b for each surface is known:

Then, the problem reduces to determining the radiosities J_1, J_2 and J_3.

This is done by applying Kirchoff's law of d.c. circuits to each node: i.e. sum of the currents (or, rate of heat transfers) entering *into* each node is zero:

$$\text{Node } J_1: \quad \frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0 \quad \dots\dots(13.63,a)$$

$$\text{Node } J_2: \quad \frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0 \quad \dots\dots(13.63,b)$$

$$\text{Node } J_3: \quad \frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0 \quad \dots\dots(13.63,c)$$

Solving these three eqns. simultaneously, we get J_1, J_2 and J_3.

Once the radiosities are known, expressions for net heat flows between the surfaces are:

$$Q_{12} = \frac{J_1 - J_2}{R_{12}} = \frac{J_1 - J_2}{\dfrac{1}{A_1 \cdot F_{12}}} \qquad ....(13.64,a)$$

$$Q_{13} = \frac{J_1 - J_3}{R_{13}} = \frac{J_1 - J_3}{\dfrac{1}{A_1 \cdot F_{13}}} \qquad ....(13.64,b)$$

$$Q_{23} = \frac{J_2 - J_3}{R_{23}} = \frac{J_1 - J_2}{\dfrac{1}{A_2 \cdot F_{23}}} \qquad ....(13.64,c)$$

And, net heat flow from each surface is:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{E_{b1} - J_1}{\left(\dfrac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1}\right)} \qquad \dots(13.65,a)$$

$$Q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{E_{b2} - J_2}{\left(\dfrac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}\right)} \qquad \dots(13.65,b)$$

$$Q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{E_{b3} - J_3}{\left(\dfrac{1 - \varepsilon_3}{A_3 \cdot \varepsilon_3}\right)} \qquad \dots(13.65,c)$$

To the general eqns (13.64), apply the constraints such as:

If the surface is black or re-radiating: $J_i = E_{bi} = \sigma \cdot T_i^4$.

And, $Q_i = 0$ for a re-radiating surface.

If Q_i at any surface is specified instead of the temperature (i,e. E_{bi}), then, $(E_{bi} - J_i)/R_i$ is replaced by Q_i.

Few special cases of three-zone enclosures:

    1. **Two black surfaces connected to a third refractory surface:**

Fig.(a)

Fig.(b)
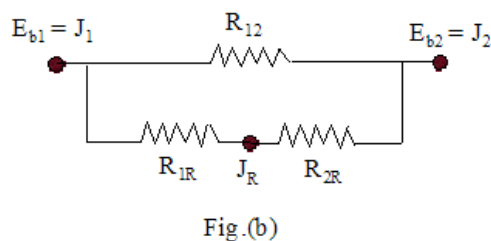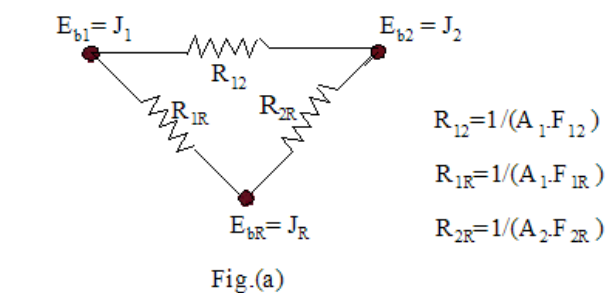
Figure 13.33 Two black surfaces connected by a thrid re-radiating surface and its radiation network

Fig. (13.33,a) shows the radiation network for this case. The radiation network is drawn very easily by *remembering the usual principles*: for a black surface, the surface resistance is zero, i.e. $E_b = J$. For a re-radiating surface too, $E_b = J$, as already explained; further, for a re-radiating surface, $Q = 0$. Between two given surfaces, the radiosity potentials are connected by the respective space resistances, as shown. It may be observed that the system reduces to a series-parallel circuit of resistances as shown in Fig. (13.33,b).

We get:

$$Q_{12} = \sigma \cdot \left(T_1^4 - T_2^4\right) \cdot \left[A_1 \cdot F_{12} + \cfrac{1}{\left(\cfrac{1}{A_1 \cdot F_{1R}} + \cfrac{1}{A_2 \cdot F_{2R}}\right)} \right] \qquad \text{.....(13.66)}$$

Here, Q_{12} is the net radiant heat transferred between surfaces 1 and 2. Similar expressions can be written for heat transfer between surfaces 2 and 3 (= Q_{23}) and the heat transfer between surfaces 1 and 3 (= Q_{13}).

----

2. **Two gray surfaces surrounded by a third re-radiating surface:**
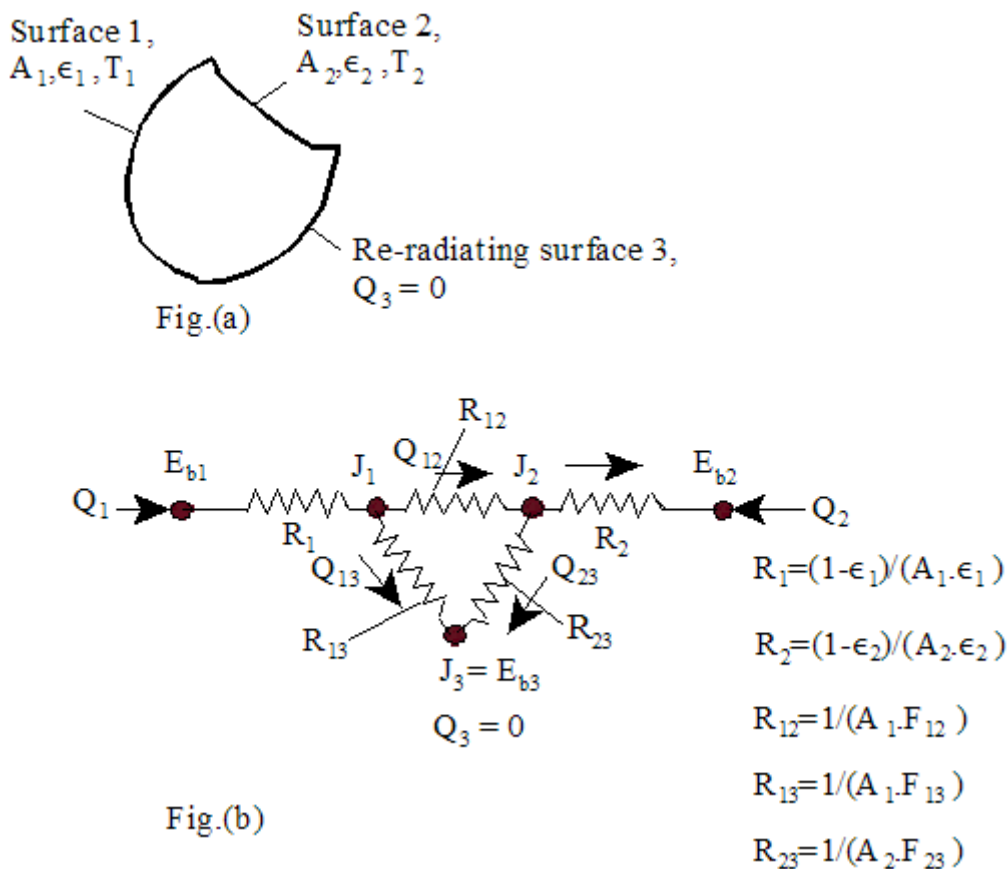
Figure 13.34 Two gray surfaces surrounded by a re-radiating surface

Here, we have:

$$Q_3 = 0$$

i.e.
$$\frac{E_{b3} - J_3}{\left(\dfrac{1 - \varepsilon_3}{A_3 \cdot \varepsilon_3}\right)} = 0$$

i.e.
$$E_{b3} = J_3$$

i.e. once the radiosity of the re-radiating surface is known, its temperature can easily be calculated, since $E_{b3} = \sigma \cdot T_3^4$. Further, note that T_3 is independent of the emissivity of surface 3.

Expression for heat flow rate is:

$$Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_{tot}}$$

where, R_{tot} is the total resistance, given by:

$$R_{tot} = R_1 + \left[\frac{1}{\dfrac{1}{R_{12}} + \dfrac{1}{(R_{13} + R_{23})}}\right] + R_2$$

i.e.
$$R_{tot} = \left(\frac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1}\right) + \frac{1}{\left[A_1 \cdot F_{12} + \dfrac{1}{\left(\dfrac{1}{A_1 \cdot F_{13}} + \dfrac{1}{A_2 \cdot F_{23}}\right)}\right]} + \left(\frac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}\right) \qquad ....(13.67)$$

Radiation heat exchange in *four zone enclosures***:**

a) **When all the four surfaces are black:** Remembering the principles already explained, if the radiation network for an enclosure comprising of four black surfaces is drawn, it will look as shown in Fig. (13.35), below:
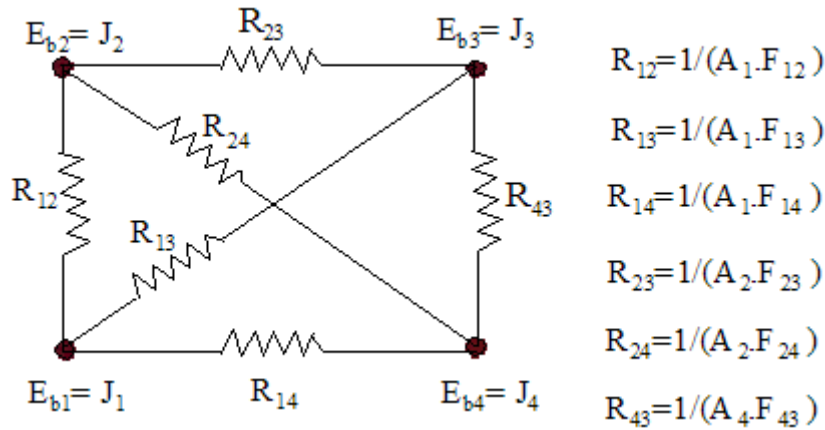
Fig 13.35 Radiation network for an enclosure of four black surfaces

The equations shown in the figure are:

$$R_{12} = 1/(A_1 \cdot F_{12})$$
$$R_{13} = 1/(A_1 \cdot F_{13})$$
$$R_{14} = 1/(A_1 \cdot F_{14})$$
$$R_{23} = 1/(A_2 \cdot F_{23})$$
$$R_{24} = 1/(A_2 \cdot F_{24})$$
$$R_{43} = 1/(A_4 \cdot F_{43})$$

Expression for net radiant heat flow rate from surface 1 is:

$$Q_1 = \frac{J_1 - J_2}{R_{12}} + \frac{J_1 - J_3}{R_{13}} + \frac{J_1 - J_4}{R_{14}}$$

i.e.

$$Q_1 = \frac{E_{b1} - E_{b2}}{R_{12}} + \frac{E_{b1} - E_{b3}}{R_{13}} + \frac{E_{b1} - E_{b4}}{R_{14}}$$ ...since $E_{b1} = J_1$ ec.

i.e.

$$Q_1 = A_1 \cdot F_{12} \cdot (E_{b1} - E_{b2}) + A_1 \cdot F_{13} \cdot (E_{b1} - E_{b3}) + A_1 \cdot F_{14} \cdot (E_{b1} - E_{b4})$$ ....(13.68)

Similar expressions can be written for the net heat flow from other three surfaces.

b) **When all the four surfaces are gray:** Now, for each surface, a surface resistance has also to be included, and the radiation network for this system will be as shown in Fig. (13.36):
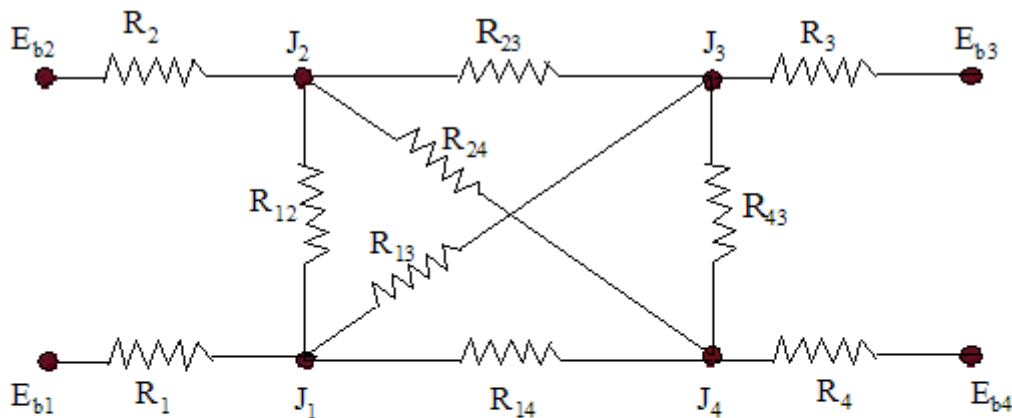
$$R_1 = (1-\epsilon_1)/(A_1 \cdot \epsilon_1) \qquad R_{12} = 1/(A_1 \cdot F_{12}) \qquad R_{23} = 1/(A_2 \cdot F_{23})$$

$$R_2 = (1-\epsilon_2)/(A_2 \cdot \epsilon_2) \qquad R_{13} = 1/(A_1 \cdot F_{13}) \qquad R_{24} = 1/(A_2 \cdot F_{24})$$

$$R_3 = (1-\epsilon_3)/(A_3 \cdot \epsilon_3) \qquad R_{14} = 1/(A_1 \cdot F_{14}) \qquad R_{43} = 1/(A_4 \cdot F_{43})$$

$$R_4 = (1-\epsilon_4)/(A_4 \cdot \epsilon_4)$$

Fig 13.36 Radiation network for an enclosure of four gray surfaces

Expression for net radiant heat flow rate from surface 1 is:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{J_1 - J_2}{R_{12}} + \frac{J_1 - J_3}{R_{13}} + \frac{J_1 - J_4}{R_{14}}$$

i.e.

$$Q_1 = \frac{E_{b1} - J_1}{\left(\frac{1-\epsilon_1}{A_1 \cdot \epsilon_1}\right)} = A_1 \cdot F_{12} \cdot (J_1 - J_2) + A_1 \cdot F_{13} \cdot (J_1 - J_3) + A_1 \cdot F_{14} \cdot (J_1 - J_4)$$ ....(13.69)

Similar expressions can be written for the net heat flow from other three surfaces.

Radiation shielding:

For two infinite, parallel plates 1 and 2, radiation shield 3, be placed between the plates:
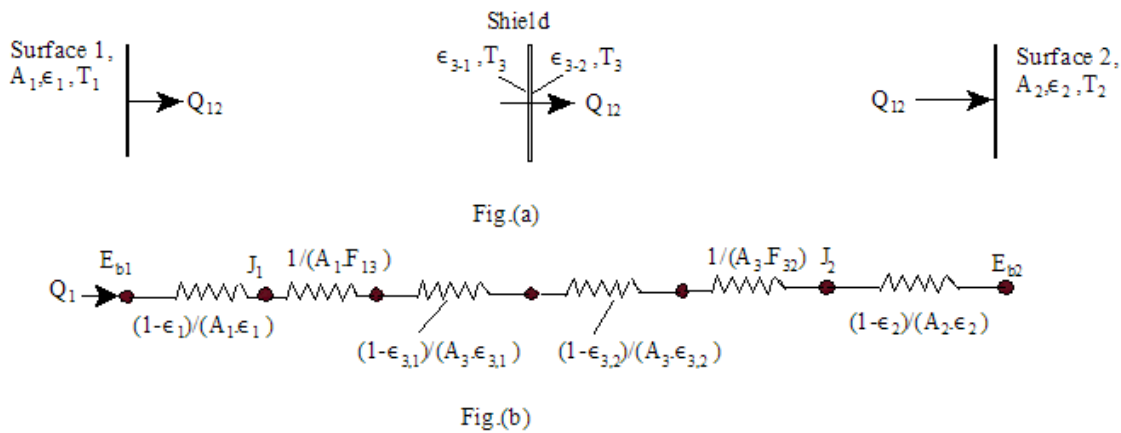
Fig 13.37 Radiation shield between two parallel plates, and associated radiation network

When there is no shield, the radiation heat transfer between plates 1 and 2 is already shown to be:

$$Q_{12} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \quad \text{....for infinitely large parallel plates.....(13.59)}$$

With one shield placed between plates 1 and 2, the radiation network will be as shown in Fig. (b) above. Note that now all the relevant resistances are in series. Net heat transfer between plates 1 and 2 is given as:

$Q_{12\text{-one shield}} = (E_{b1} - E_{b2})/R_{tot}$ where R_{tot} is the total resistance.

i.e.

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \dfrac{1}{A_1 \cdot F_{13}} + \dfrac{1-\varepsilon_{3\_1}}{A_3 \cdot \varepsilon_{3\_1}} + \dfrac{1-\varepsilon_{3\_2}}{A_3 \cdot \varepsilon_{3\_2}} + \dfrac{1}{A_3 \cdot F_{32}} + \dfrac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

....for two gray surfaces with one radiation shield
placed in between.....(13.70)

Now, for two large parallel plates, we note:

$$F_{13} = F_{32} = 1 \quad \text{and,} \quad A_1 = A_2 = A_3 = A$$

Then, eqn. (13.70) simplifies to:

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3\_1}} + \dfrac{1}{\varepsilon_{3\_2}} - 1\right)} \qquad ....(13.71)$$

Note that as compared to eqn. (13.59) for the case of no-shield, we have, **with one shield**, *an additional term appearing in the denominator* of eqn. (13.71). Therefore, if there are N radiation shields, we have, for net radiation heat transfer:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon3\_1} + \dfrac{1}{\varepsilon_{3\_2}} - 1\right) + ... + \left(\dfrac{1}{\varepsilon_{N\_1}} + \dfrac{1}{\varepsilon_{N\_2}} - 1\right)} \qquad ...(13.72)$$

If emissivities of all surfaces are equal, eqn. (13.72) becomes:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{(N+1) \cdot \left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)} = \frac{1}{(N+1)} \cdot \left(Q_{12_{no\_shield}}\right) \qquad ....(13.73)$$

Note this important result, which implies that, *when all emissivities are equal, presence of one radiation shield reduces the radiation heat transfer between the two surfaces to one-half*, two radiation shields reduce the heat transfer to one-third, 9 radiation shields reduce the heat transfer to one-tenth etc.

For a *more practical case of the two surfaces having emissivities of ε1 and ε2, and all shields having the same emissivity of εs,* eqn. (13.72) becomes:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot \left(T_1^{\,4} - T_2^{\,4}\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N \cdot \left(\dfrac{2}{\varepsilon_s} - 1\right)} \qquad ....(13.74)$$

To determine the equilibrium temperature of the radiation shield:

Once Q_{12} is determined from eqn. (13.71), the temperature of the shield is easily found out by applying the condition that in steady state:

$$Q_{12} = Q_{13} = Q_{32} \qquad ...(13.75)$$

We can use either of the conditions: $Q_{12} = Q_{13}$ or $Q_{12} = Q_{32}$.

Q_{13} or Q_{32} is determined by applying eqn. (13.59); i.e. we get:

$$Q_{12} = Q_{13} = \frac{A \cdot \sigma \cdot \left(T_1^{\,4} - T_3^{\,4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1} \qquad ...(13.76,a)$$

or,

$$Q_{12} = Q_{32} = \frac{A \cdot \sigma \cdot \left(T_3^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1} \qquad ....(13.76,b)$$

In both the above eqns. T_3 is the only unknown, which can easily be determined.

For a cylindrical radiation shield placed in between two, long concentric cylinders:

The radiation heat transfer between two long, concentric cylinders is:

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \left(\dfrac{A_1}{A_2}\right) \cdot \left(\dfrac{1}{\varepsilon_2} - 1\right)}$$ ....for infinitely long concentric cylinders.....(13.60)

where

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Now, let a cylindrical radiation shield, 3, be placed in between the inner cylinder (1) and the outer cylinder (2), as shown in Fig.(13.38).
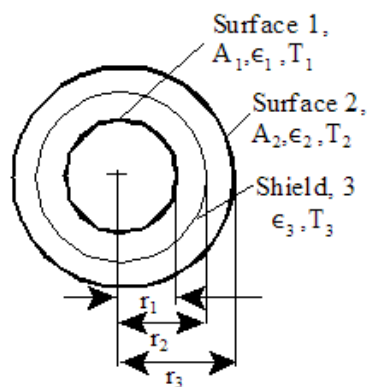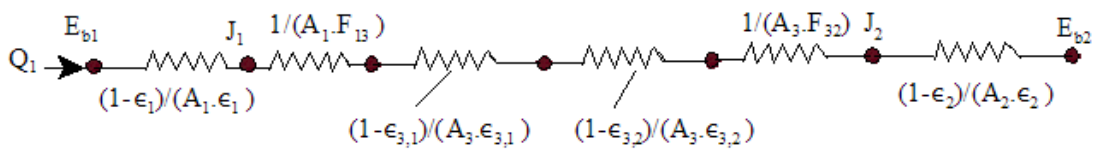
Fig 13.38 Radiation shield between two concentric cylinders, and associated radiation network

The radiation network for this system is shown in Fig. (13.38,b) and it is exactly the same as shown in Fig. (13.37,b).

And, the radiation heat transfer between cylinders 1 and 2, when the shield is present, is given by:

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{13}} + \frac{1 - \varepsilon_{3\_1}}{A_3 \cdot \varepsilon_{3\_1}} + \frac{1 - \varepsilon_{3\_2}}{A_3 \cdot \varepsilon_{3\_2}} + \frac{1}{A_3 \cdot F_{32}} + \frac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

....for two gray surfaces with one radiation shield
placed in between.....(13.70)

Now, for the cylindrical system, we have:

$$F_{13} = F_{32} = 1$$

$$A_1 = 2 \cdot \pi \cdot r_1 \cdot L$$

$$A_2 = 2 \cdot \pi \cdot r_2 \cdot L$$

and,

$$A_3 = 2 \cdot \pi \cdot r_3 \cdot L$$

Then, eqn. (13.70) reduces to:

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

...for concentric cylinders with one radiation shield...(13.77)

In eqn. (13.77), we have: $(A_1/A_2) = (r_1/r_2)$, and $(A_1/A_3) = (r_1/r_3)$.

Note that as compared to the relation for two concentric cylinders with no shield (i.e. eqn. 13.60), *an additional term appears in the denominator of eqn. (13.77)* (i.e. the third term) when one radiation shield is introduced; if there is a second radiation shield, say (4), then one more similar term will have to be added in the denominator to take care of the resistance of that shield.

In this case too, the **equilibrium temperature of the shield is determined** by applying the principle that, in steady state,

$$Q_{12} = Q_{13} = Q_{32}$$

For a spherical radiation shield placed in between two concentric spheres:

This case is also represented by Fig. (13.38,a), where inner sphere 1 is enclosed by an outer sphere 2, and a radiation shield 3, is placed in between. The radiation network for this system is shown in Fig. (13.38,b).

When there is no radiation shield, radiation heat transfer between surfaces 1 and 2 is given by eqn. (13.61), viz.

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad \text{....for concentric spheres.....(13.61)}$$

where

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Again, when the radiation shield is present, the general relation for radiation heat transfer between surfaces 1 and 2 is eqn. (13.70). Remembering that for concentric spheres,

$$F_{13} = F_{32} = 1$$

$$A_1 = 4 \cdot \pi \cdot r_1^2$$

$$A_2 = 4 \cdot \pi \cdot r_2^2$$

and,
$$A_3 = 4 \cdot \pi \cdot r_3^2$$

Relation for radiant heat transfer between surfaces 1 and 2, is exactly as eqn. (13.77), i.e.

$$Q_{12_{one\_shield}} = \frac{A_1 \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \left(\dfrac{A_1}{A_2}\right) \cdot \left(\dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{A_1}{A_3}\right) \cdot \left(\dfrac{1}{\varepsilon_{3\_1}} + \dfrac{1}{\varepsilon_{3\_2}} - 1\right)}$$

...for concentric spheres with one radiation shield...(13.78)

In eqn. (13.78), we have:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \qquad \text{and,} \qquad \frac{A_1}{A_3} = \left(\frac{r_1}{r_3}\right)^2$$

In this case also, **equilibrium temperature of the shield is determined** by applying the principle that, in steady state,

$$Q_{12} = Q_{13} = Q_{32}$$

Radiation error in temperature measurement:
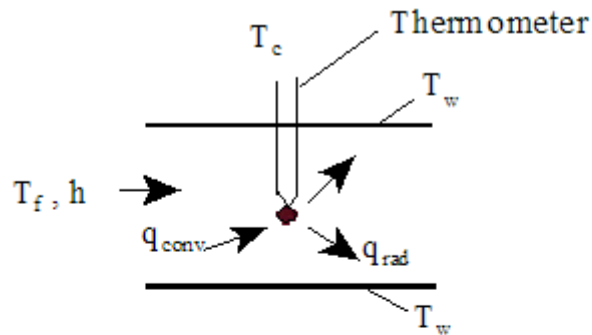
(a) Thermometer without radiation shield
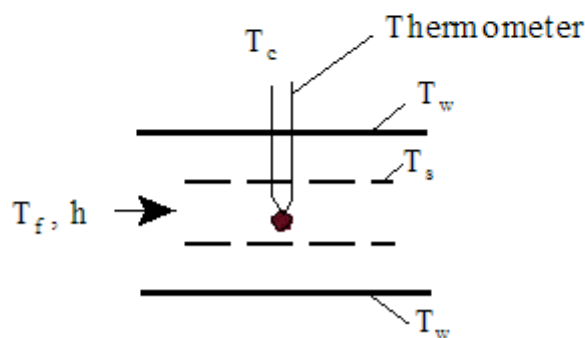
(b) Thermometer with radiation shield

Fig 13.39 Radiation shielding of thermometers

Let the reading *shown* by the thermometer be T_c. This reading, however, does not represent the true temperature of the fluid T_f, since the thermometer bulb will lose heat by radiation to the walls of the channel which are at a lower temperature T_w (which is usually the case).

The temperature T_c shown by the thermometer will be some value in between T_f and T_w.

We wish to find out the true temperature of the fluid T_f, by knowing the thermometer reading T_c.

Making an **energy balance on the thermometer bulb,** in steady state, we have:

Without radiation shield:

q_{conv} to the bulb = q_{rad} from the bulb

i.e. $\quad h \cdot A_c \cdot (T_f - T_c) = \varepsilon_c \cdot A_c \cdot \sigma \cdot (T_c^4 - T_w^4)$

i.e. $\quad T_f = T_c + \dfrac{\varepsilon_c \cdot \sigma \cdot (T_c^4 - T_w^4)}{h} \qquad \text{.....(13.79)}$

where A_c = surface area of thermometer bulb,

ε_c = emissivity of thermometer bulb surface

Eqn. (13.79) gives the true temperature of the fluid T_f.

Second term on the RHS of eqn. (13.79) represents the *error in temperature measurement* due to **radiation effect.**

Energy balance on the *thermometer bulb*:

Heat transferred to the bulb from the fluid by convection =
Heat transferred from the bulb to the shield by radiation

i.e.
$$h \cdot A_c \cdot \left(T_f - T_c \right) = \frac{\sigma \cdot \left(T_c^{\ 4} - T_s^{\ 4} \right)}{\left(\dfrac{1 - \varepsilon_c}{A_c \cdot \varepsilon_c} \right) + \dfrac{1}{A_c \cdot F_{cs}} + \left(\dfrac{1 - \varepsilon_s}{A_s \cdot \varepsilon_s} \right)} \quad ....(13.80)$$

In eqn. (13.80), F_{cs} = view factor of thermometer bulb w.r.t the shield and is, generally equal to 1.

Making an energy balance on the *shield*:

heat transferred to shield from the fluid by convection +
heat transferred to shield from bulb by radiation =
heat transferred from shield to walls by radiation

i.e.
$$2 \cdot A_s \cdot h \cdot \left(T_f - T_s \right) + \frac{\sigma \cdot \left(T_c^{\ 4} - T_s^{\ 4} \right)}{\left(\dfrac{1 - \varepsilon_c}{A_c \cdot \varepsilon_c} \right) + \dfrac{1}{A_c \cdot F_{cs}} + \left(\dfrac{1 - \varepsilon_s}{A_s \cdot \varepsilon_s} \right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot \left(T_s^{\ 4} - T_w^{\ 4} \right) \quad ....(13.81)$$

where, A_s = area of shield on one side

ε_s = emissivitty of shield surface

A_c = area of bulb surface

ε_c = emissivity of bulb surface

F_{cs} = view factor of bulb w.r.t. shield

In the first term of the above eqn. factor 2 appears since convective heat transfer to the shield occurs on *both surfaces* of the shield. Also, in writing the RHS, the inherent assumption is that:
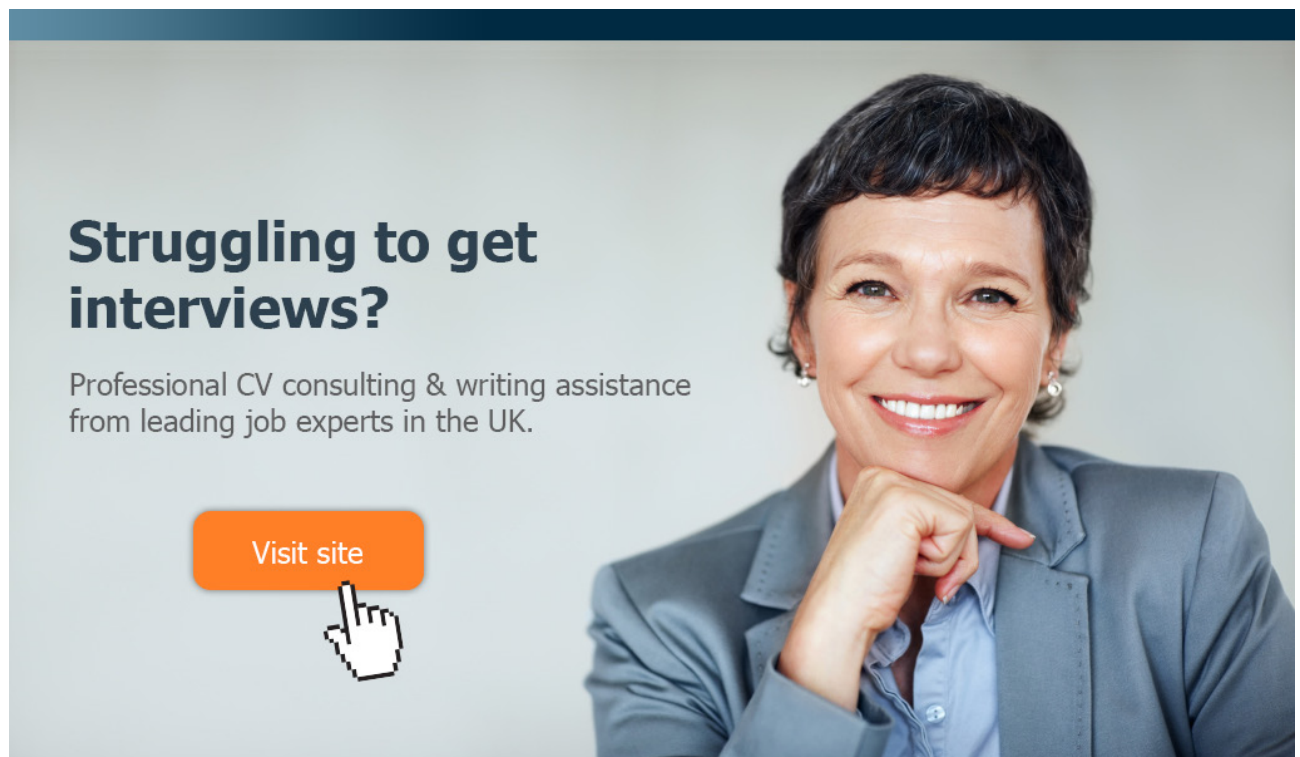
$$F_{sw} = 1 \quad \dots \text{view factor between the shield and the walls}$$

and,

$$\frac{A_s}{A_w} = 0 \quad \dots \text{i.e. surface area of shield is negligible compared to the area of the channel walls}$$

Solving eqns. (13.80) and (13.81) simultaneously, we obtain the shield temperature T_s and the thermometer reading T_c, (if T_f is known), or T_f (if T_c is known).

==

5A Laws of Radiation and Radiation properties:

Prob.5A.1: Draw a plot of Planck's Law for spectral distribution:

Mathcad Solution:

Planck's Law:

$$E_{b\lambda} = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1} \qquad ...W/(m^2.\mu m).....(13.7)$$

where:

We have, costants C_1 and C_2 in Planck's Law:

$$C_1 := 3.742 \cdot 10^8 \quad W.\mu m^4/m^2$$

$$C_2 := 1.4387 \cdot 10^4 \quad \mu m.K$$

To plot the Planck's disribution for a black body, using Mathcad, **first, define $E_{b\lambda}$ as a function of T and λ as follows:**

$$E_{b\lambda}(\lambda, T) := \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1} \qquad ....E_{b\lambda} \text{ as a function of } \lambda \text{ and T}$$

To draw the plot:

Define a range variable λ varying from 0.01 µm o 1000 µm:

$$\lambda := 0.01, 0.02.. 1000 \qquad ....\text{define the range variable}$$

Now, select the x-y plot from the graph palette; on x-axis place holder, type λ and on the y-axis place holder, fill in $E_{b\lambda}$ (λ,100), $E_{b\lambda}$ (λ,500), $E_{b\lambda}$ (λ,1000),and $E_{b\lambda}$ (λ,5800). Click anywhere outside the graph region, and the curves appear immediately:

Spectral Emissive Power of a Black body

Legend:
- T = 100 K
- T = 500 K
- T = 1000 K
- T = 5800 K = temp. of Sun
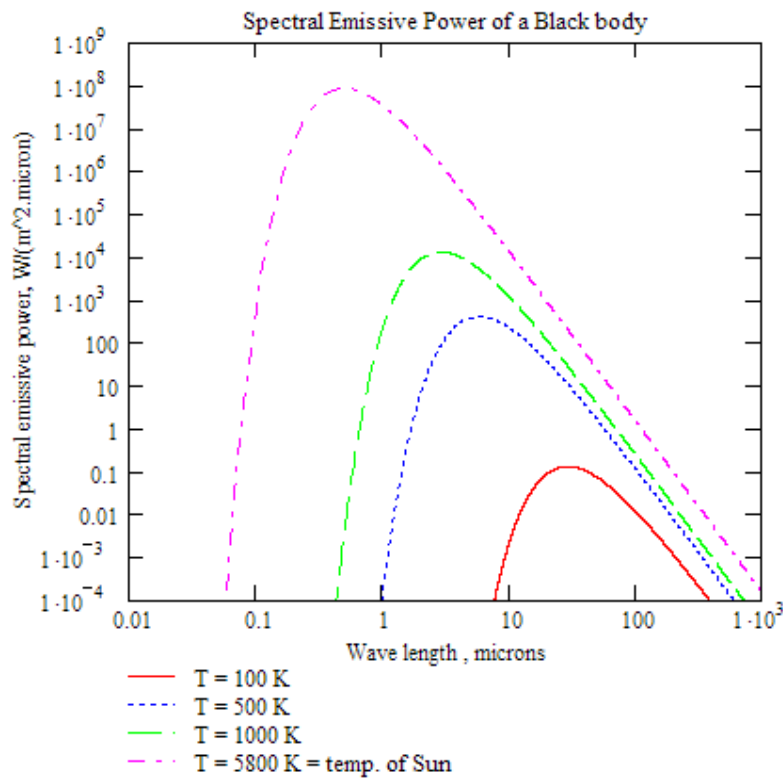
See the ease with which this plot is drawn with Mathcad.

==

Prob.5A.2. Radiation strikes an object with transmissivity of 0.03 and reflectivity of 0.5. The absorbed flux is indirectly measured to be 94 W/m2. Compute the incident flux. **(M.U.)**

Mathcad Solution:

Data:

$$\tau := 0.03 \quad \text{transmissivity}$$

$$\rho := 0.5 \quad \text{reflectivity}$$

$$Q_a := 94 \quad \text{W/m}^2 \ldots \text{absorbed flux}$$

Let G = incident flux

Now, we have: $\alpha + \tau + \rho = 1$ where, $\alpha = Q_a/G$

Therefore,

$$\alpha = \frac{Q_a}{G} = 1 - \tau - \rho$$

i.e. $G := \dfrac{Q_a}{1 - \tau - \rho}$

i.e. $G = 200$ W/m^2....Incident flux....Ans.

===

Prob.5A.3. A hole of area dA = 2 cm2 is opened on the surface of a large spherical cavity whose inside is maintained at 1000 K. Calculate: (a) the radiation energy streaming through the hole in all directions into space, (b) the radiation energy streaming per unit solid angle in a direction making a 60 deg. angle with the normal to the surface of the opening. **(M.U.)**

Mathcad Solution:
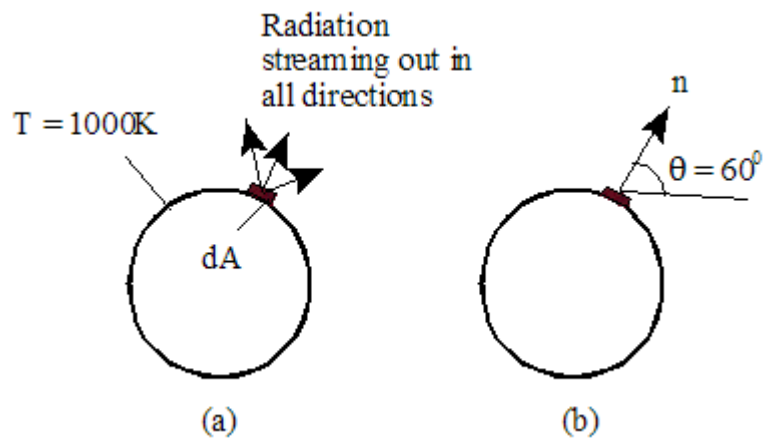
Data:

$$dA := 2 \cdot 10^{-4} \quad m^2 \ldots\text{area of the hole}$$

$$T := 1000 \quad K\ldots\text{temperature}$$

$$\sigma := 5.67 \cdot 10^{-8} \quad W/(m^2.K^4)\ldots\text{Stefan-Boltzann constant}$$

$$\theta = 60 \, \text{deg.} \quad \ldots\text{angle made by the desired direction, with the normal to the surface}$$

i.e. $\theta := 60 \cdot \dfrac{\pi}{180}$ ..radians

(a) Radiation streaming out in all directions:

Since the spherical cavity can be considered as a black body, energy streaming out is given by Stefan-Boltzmann law:

i.e. $Q := dA \cdot \sigma \cdot T^4$

or, $Q = 11.34$ **W...Radiaion energy streaming through the hole...Ans.**

(b) Radiation streaming out through unit solid angle, in a direction making 60 deg. with normal:

Now, we have the relation: $E_b = \pi \cdot I_b$, where E_b = Emissive power, and I_b = Intensity of radiation.

Therefore, $\quad I_b = \dfrac{E_b}{\pi}$

But, $\quad E_b = \sigma \cdot T^4 \quad$ W/m² ...for a black body, by Stefan-Boltzmann Law

i.e. $\quad I_b := \dfrac{\sigma \cdot T^4}{\pi} \quad$ W/(m².sr)....Intensity of radiation

i.e. $\quad I_b = 1.805 \times 10^4 \quad$ W/(m².sr)

And, radiation streaming out in that direction:

$Q := I_b \cdot dA \cdot \cos(\theta) \quad$ W....where θ is in radians (**Note:** while using Mathcad, θ must be in radians while calculating cos(θ).)

i.e. $\quad Q = 1.805 \quad$ **W...radiaion through a solid angle of unity, in a direction of 60 deg. with normal to the surface...Ans.**

==

Prob.5A.4. It is observed that intensity of radiation is maximum in case of solar radiation at a wavelength of 0.49 microns. Assuming the Sun as a black body, estimate its surface temperature and emissive power. Wein displacement constant = 0.289 × 10-2 m.K. **(M.U.)**

Mathcad Solution:

Data:

$\lambda_{max} := 0.49 \quad$ microns...wave length of max. radiation

$\sigma := 5.67 \cdot 10^{-8} \quad$ W/(m².K⁴)....Stefan-Boltzmann const.

By Wein's displacement law, we have:

$$\lambda_{max} \cdot T = 2890 \quad \text{microns.K}$$

Therefore,

$$T := \frac{2890}{\lambda_{max}}$$

i.e. $T = 5.898 \times 10^3$ K...surface temp. of Sun...Ans.

Heat flux at the surface, E_b:

Sun an be considered as a black body; then, from Stefan-Boltzmann Law:

$$E_b := \sigma \cdot T^4$$

i.e. $E_b = 6.861 \times 10^7$ W/m²...heat flux at the surface of Sun...Ans.

==

Prob.5A.5. The temperature of a body of area 0.1m2 is 900 K. Calculate the total rate of energy emission, intensity of normal radiation in W/(m2.sr), maximum monochromatic emissive power, and wavelength at which it occurs. **(M.U.)**

Mathcad Solution:

Data:

$T := 900$ K....temperature

$A := 0.1$ m²...area

$\sigma := 5.67 \cdot 10^{-8}$ W/(m².K⁴)....Stefan-Boltzmann const.

Total rate of energy emission:

$$E_b := \sigma \cdot T^4 \quad \text{..from Stefan-Boltzmann Law}$$

i.e. $E_b = 3.72 \times 10^4$ W/m²....Total emissive power

Therefore,

$$Q := E_b \cdot A$$

i.e. $Q = 3.72 \times 10^3$ W...Total energy emission from surface...Ans.

Intensity of normal radiation:

(Note that for a black (diffuse) surface, intensity is the same in all directions.)

$$I := \frac{E_b}{\pi}$$

i.e. $\quad I = 1.184 \times 10^{4} \quad$ W/(m^2.sr).....Ans.

Wave length of max. monochromatic emissive power:

From Wein's displacement law, we have:

$$\lambda_m \cdot T = 2898 \quad \mu m.K$$

Therefore,

$$\lambda_m := \frac{2898}{T} \quad \mu m$$

i.e. $\quad \lambda_m = 3.22 \quad \mu m$....wave length for max. monochromatic emissive power at 900 K...Ans
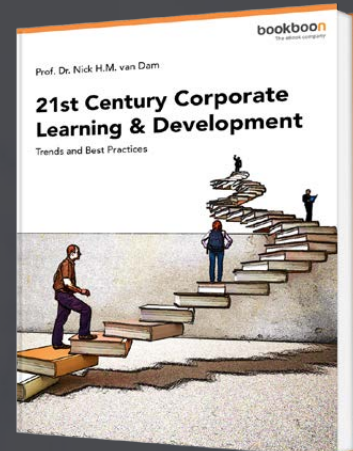
Max. monochromatic emissive power:

We use Planck's law, with values of constants, C1 and C2:

$$C1 := 3.742 \cdot 10^8 \quad W.\mu m^4/m^2)$$

and, $\quad C2 := 1.4387 \cdot 10^4 \quad \mu m.K$

$$E_{b\lambda.max} := \frac{C1 \cdot \lambda_m^{-5}}{\exp\left(\dfrac{C2}{\lambda_m \cdot T}\right) - 1} \quad ....\text{Planck's Law}$$

i.e. $\quad E_{b\lambda.max} = 7.6 \times 10^3 \qquad W/(m^2.micron)...Max.$ monochr. emissive
power....Ans.

Alternatively:

We can directly apply eqn.(13.12):

$$E_{b\lambda.m} := 1.287 \cdot 10^{-5} \cdot T^5 \quad W/m^3.....(13.12)$$

i.e. $\quad E_{b\lambda.m} = 7.6 \times 10^9 \quad W/m^3 = 7.6 \times 10^{-3}$ W/(m^2.μm)..same as
obtained above...Ans.

===

Prob.5A.6. Write a Mathcad program to read the Radiation Functions (i.e. radiation emitted in a wave band as a fraction of Total emissive power) against λT from the Table 13.2 and return its value at the desired λT by interpolation:

We have:

$$E_{b\lambda}(\lambda, T) := \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1}$$

$$F_{0\_\lambda} = \frac{\displaystyle\int_0^\lambda E_{b\lambda}\, d\lambda}{\sigma \cdot T^4} = \frac{\displaystyle\int_0^{\lambda \cdot T} E_{b\lambda}\, d(\lambda \cdot T)}{\sigma \cdot T^5}$$

First, write the Table 13.2 for F_0_λ vs λT in two columns only, and then be set it equal to Matrix M1. First column of the Matrix is λT values and the second column is F_0_λ values.

Then:

$$F_{0\_\lambda}(M1, \lambda T) := \begin{vmatrix} (\text{return } 0) & \text{if } \lambda T \leq 600 \\ (\text{return } 1) & \text{if } \lambda T > 100000 \\ \text{LambdaT\_vector} \leftarrow M1^{\langle 0 \rangle} \\ \text{F\_vector} \leftarrow M1^{\langle 1 \rangle} \\ \text{F\_0\_}\lambda \leftarrow \text{linterp}(\text{LambdaT\_vector}, \text{F\_vector}, \lambda T) \end{vmatrix}$$

Note that F_0_λ = 0 for λT < 600, and F_0_λ = 1 for λT >100000 (See the Table 13.2 at the beginning of this Chapter).

Ex: $\lambda T := 12000$

$$F_{0\_\lambda}(M1, \lambda T) = 0.9451$$

Also, if we need to get fraction between two given wave lengths λ1 and λ2:

$$F_{\lambda 1\_\lambda 2}(M1, \lambda 1T, \lambda 2T) := F_{0\_\lambda}(M1, \lambda 2T) - F_{0\_\lambda}(M1, \lambda 1T)$$

Ex: $\lambda 1T := 1600$ $\lambda 2T := 12000$

$$F_{\lambda 1\_\lambda 2}(M1, \lambda 1T, \lambda 2T) = 0.925$$

Let us use this Function in the following problem:

Prob.5A.7. Window glass transmits radiant energy in the wave length range 0.4μm to 2.5 μm. Determine the fraction of total radiant energy which is transmitted, when the source temperature is: (a) 5800 K (i.e. Sun's surface temp.), and (b) 300 K (i.e. room temp.)

Mathcad Solution:

Data:

$T1 := 5800$ K....source temperature in first case

$T2 := 300$ K....source temperature in second case

$\lambda_1 := 0.4$ μm...lower end of wave length band in which glass is transparent

$\lambda_2 := 2.5$ μm...upper end of wave length band in which glass is transparent

$\sigma := 5.67 \cdot 10^{-8}$ W/(m^2.K^4)....Stefan-Boltzmann const.

Case (a): Source temp. T1 = 5800 K

We use the Mathcad Functions written above, or Table 13.2 where radiation functions are tabulated against the product (λ.T).

We have:

$\lambda_1 \cdot T1 = 2.32 \times 10^3$ μm.K

$\lambda_2 \cdot T1 = 1.45 \times 10^4$ μm.K

Corresponding to these λT values, we get:

$$F_{0\_\lambda 1} := F_{0\_\lambda}\left(M1, \lambda_1 \cdot T1\right) \quad ....\text{using the Mathcad Function written above}$$

i.e. $\quad F_{0\_\lambda 1} = 0.1245$

and, $\quad F_{0\_\lambda 2} := F_{0\_\lambda}\left(M1, \lambda_2 \cdot T1\right) \quad\quad ....\text{using the Mathcad Function written above}$

i.e. $\quad F_{0\_\lambda 2} = 0.9661$

Therefore, fraction transmmitted is equal to:

$$0.9661 - 0.12454 = 0.842$$

Alternatively: $\quad F_{\lambda 1\_\lambda 2}\left(M1, \lambda_1 \cdot T1, \lambda_2 \cdot T1\right) = 0.842$

i.e. 84.2% of the energy coming from the Sun (at 5800 K) is transmitted through the window glass.....Ans.

Case (b): Source temp. T2= 300 K

Again, we use the Mathcad Functions already written or Table 13.2 where radiation functions are tabulated against the product $(\lambda.T)$.

We have:

$$\lambda_1 \cdot T2 = 120 \quad\quad\quad \mu m.K$$

$$\lambda_2 \cdot T2 = 750 \quad\quad\quad \mu m.K$$

Corresponding to these λT values, we get:

$$F_{0\_\lambda 1} := F_{0\_\lambda}\left(M1, \lambda_1 \cdot T2\right) \quad ....\text{using the Mathcad Function written above}$$

i.e. $\quad F_{0\_\lambda 1} = 0$

and, $\quad F_{0\_\lambda 2} := F_{0\_\lambda}\left(M1, \lambda_2 \cdot T2\right) \quad\quad ....\text{using the Mathcad Function written above}$

i.e. $\quad F_{0\_\lambda 2} = 1.2 \times 10^{-5} \quad = \text{almost zero}$

i.e. practically no energy will be transmitted through the window glass in this wave length range, if the source temp. = 300 K.

In other words, glass *is 'opaque'* to radiation at 300 K in the wave length range 0.4 μm to 2.5 μm.

This is the principle of 'Greenhouse effect', wherein radiation from a high temperature source (i.e. Sun) is allowed to pass through the glass into the enclosure of the green house, while radiation at a relatively low temperature from within the enclosure, is not allowed to escape out. This, in effect, causes an increase in the temperature of the space within the enclosure.

==

Prob.5A.8. Spectral emissivity of a particular surface at 800 K is approximated by a step function, as follows: $\varepsilon1 = 0.1$ for $\lambda = 0$ to 2 μm, $\varepsilon2 = 0.5$ for $\lambda = 2$ to 15 μm, and $\varepsilon3 = 0.8$ for $\lambda = 15$ to ∞. Calculate: (i) the total (hemispherical) emissive power, and (ii) total hemispherical emissivity ε, over all wave lengths.
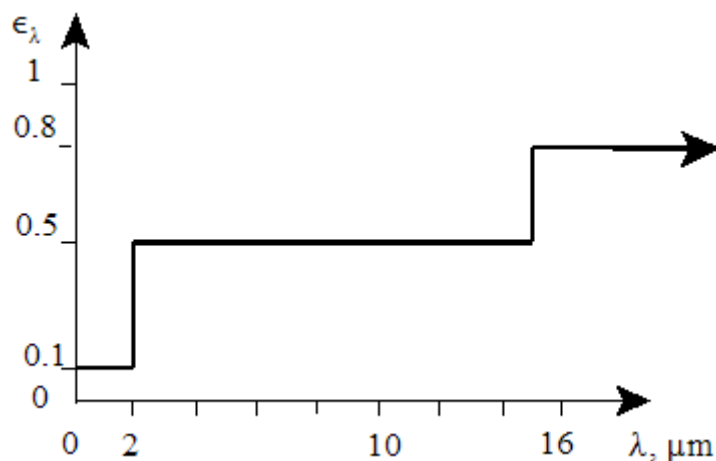
Mathcad Solution:

Data:

$T := 800$ K....temperature

$\varepsilon1 := 0.1$ ...emissiviy in wavelength range: 0 to 2 μm

$\varepsilon2 := 0.5$ ...emissiviy in wavelength range: 2 to 15 μm

$\varepsilon3 := 0.8$ ...emissiviy in wavelength range: 15 μm to ∞.

$\sigma := 5.67 \cdot 10^{-8}$ W/(m^2.K^4)....Stefan-Boltzmann const.

$\lambda_1 := 2$ μm

$\lambda_2 := 15$ μm

$\lambda_3 := \infty$

Planck's Law gives spectral emissive power of a black body.

For a non-gray surface considered in this problem, we can write:

Total emissive power: $\quad E = \displaystyle\int_0^\infty \varepsilon_\lambda(\lambda) \cdot E_{b\lambda} \, d\lambda$

Variation of ε_λ wih λ is specified in the problem.

Therefore, splitting the above integral into parts:

$$E = \int_0^2 \varepsilon_\lambda(\lambda) \cdot E_{b\lambda} \, d\lambda + \int_2^{15} \varepsilon_\lambda(\lambda) \cdot E_{b\lambda} \, d\lambda + \int_{15}^\infty \varepsilon_\lambda(\lambda) \cdot E_{b\lambda} \, d\lambda$$

i.e. $\quad E = 0.1 \cdot \displaystyle\int_0^2 E_{b\lambda} \, d\lambda + 0.5 \cdot \int_2^{15} E_{b\lambda} \, d\lambda + 0.8 \cdot \int_{15}^\infty E_{b\lambda} \, d\lambda$

Then, $\quad \varepsilon = \dfrac{E}{E_b} = \varepsilon 1 \cdot \left(F_{0\_\lambda 1}\right) + \varepsilon 2 \cdot \left(F_{\lambda 1\_\lambda 2}\right) + \varepsilon 3 \cdot \left(F_{\lambda 3\_\text{infinity}}\right)$

i.e. $\quad \varepsilon = \varepsilon 1 \cdot \left(F_{0\_\lambda 1}\right) + \varepsilon 2 \cdot \left(F_{0\_\lambda 2} - F_{0\_\lambda 1}\right) + \varepsilon 3 \cdot \left(F_{0\_\text{infinity}} - F_{0\_\lambda 2}\right) \qquad \ldots\text{(a)}$

Values of $F_{0-\lambda_1}$ etc. are obtained from the Mathcad Function written above, or from Table 13.2.

We have:

$$\lambda_1 \cdot T = 1.6 \times 10^3 \quad \text{...correspondingly, we get:}$$

$$F_{0\_\lambda 1} := F_{0\_\lambda}(M1, \lambda_1 \cdot T) \quad \text{i.e.} \quad F_{0\_\lambda 1} = 0.0197$$

And,

$$\lambda_2 \cdot T = 1.2 \times 10^4 \quad \text{...correspondingly, we get:}$$

$$F_{0\_\lambda 2} := F_{0\_\lambda}(M1, \lambda_2 \cdot T) \quad \text{i.e.} \quad F_{0\_\lambda 2} = 0.9451$$

And,

$$\lambda_3 \cdot T = \infty \quad \text{...correspondingly, we get:}$$

$$F_{0\_\lambda 3} := F_{0\_\lambda}(M1, 100001) \quad \text{....Note: For } \lambda T = \text{infinity, we have put } \lambda T > 100000.$$

i.e. $F_{0\_\lambda 3} = 1$

OR:

$$F_{0\_\lambda 3} := 1 \quad \text{...from Table 13.2}$$

Then, from eqn. (a):

$$\varepsilon := \varepsilon 1 \cdot F_{0\_\lambda 1} + \varepsilon 2 \cdot \left(F_{0\_\lambda 2} - F_{0\_\lambda 1}\right) + \varepsilon 3 \cdot \left(F_{0\_\lambda 3} - F_{0\_\lambda 2}\right)$$

i.e. $\varepsilon = 0.509$ **...Total hemisph. emissiviy over all wave lengths...Ans.**

And, total emissive power of this surface is given by:

$$E := \varepsilon \cdot \sigma \cdot T^4 \quad \text{W/m2....total emissive power}$$

i.e. $E = 1.199 \times 10^4$ **W/m2....total emissive power...Ans.**

Plot the variation of total emissivity ε and total emissive power E as Temp T varies from 500 K to 2000 K:

First, write the relevant quantities as functions of T:

$$F_{0\_\lambda 1}(T) := F_{0\_\lambda}\left(M1, \lambda_1 \cdot T\right) \qquad \text{check:} \quad F_{0\_\lambda 1}(T) = 0.0197$$

$$F_{0\_\lambda 2}(T) := F_{0\_\lambda}\left(M1, \lambda_2 \cdot T\right) \qquad \text{check:} \quad F_{0\_\lambda 2}(T) = 0.9451$$

$$F_{0\_\lambda 3} := F_{0\_\lambda}(M1, 100001) \qquad ....\text{Note: For } \lambda T = \text{infinity, we have:} \quad F_{0\_\lambda 3} = 1$$

$$\varepsilon(T) := \varepsilon 1 \cdot F_{0\_\lambda 1}(T) + \varepsilon 2 \cdot \left(F_{0\_\lambda 2}(T) - F_{0\_\lambda 1}(T)\right) + \varepsilon 3 \cdot \left(F_{0\_\lambda 3} - F_{0\_\lambda 2}(T)\right)$$

$$...\text{total hemisph. emissivity}$$

$$\text{check:} \quad \varepsilon(T) = 0.509$$

$$E(T) := \varepsilon(T) \cdot \sigma \cdot T^4 \qquad \text{W/m}^2....\text{total emissive power} \quad \text{check:} \quad E(T) = 1.181 \times 10^4$$

To plot the results:

$$T := 500, 510.. 2000 \qquad ...\text{define a range variable T}$$

T =	ε(T) =	E(T) =
500	0.548	$1.943 \cdot 10^3$
600	0.532	$3.91 \cdot 10^3$
700	0.52	$7.076 \cdot 10^3$
800	0.509	$1.181 \cdot 10^4$
900	0.497	$1.847 \cdot 10^4$
1000	0.483	$2.737 \cdot 10^4$
1100	0.467	$3.876 \cdot 10^4$
1200	0.45	$5.287 \cdot 10^4$
1300	0.431	$6.987 \cdot 10^4$
1400	0.413	$8.991 \cdot 10^4$
1500	0.394	$1.131 \cdot 10^5$
1600	0.376	$1.395 \cdot 10^5$
1700	0.358	$1.693 \cdot 10^5$
1800	0.341	$2.027 \cdot 10^5$
1900	0.324	$2.397 \cdot 10^5$
2000	0.309	$2.804 \cdot 10^5$

Plot:
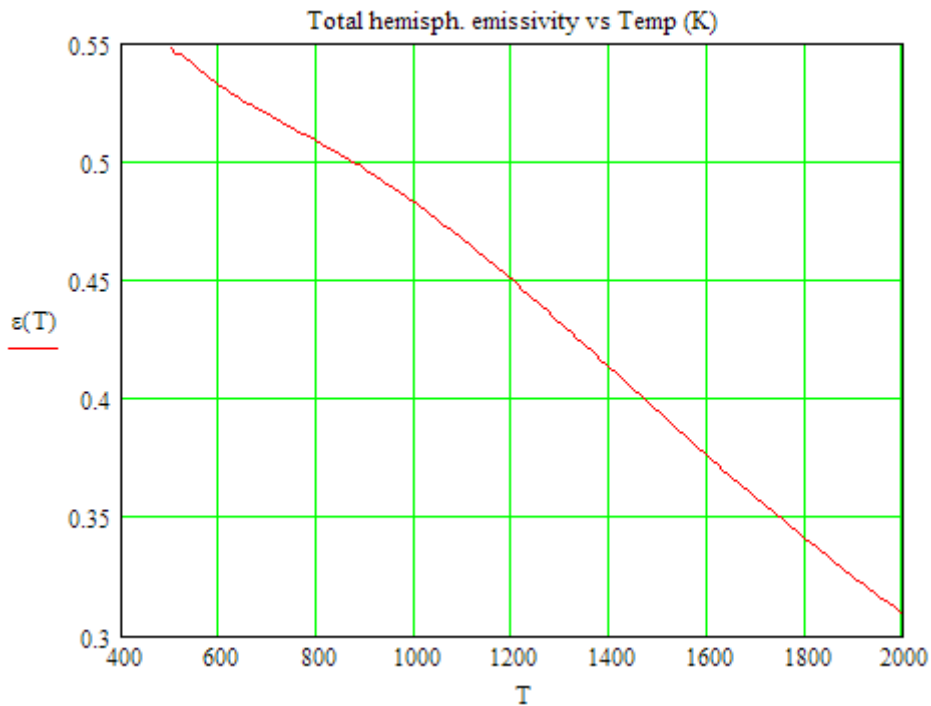
Total hemisph. emissivity vs Temp (K)

Total Emissive power (W/m^2) vs Temp (K)

==
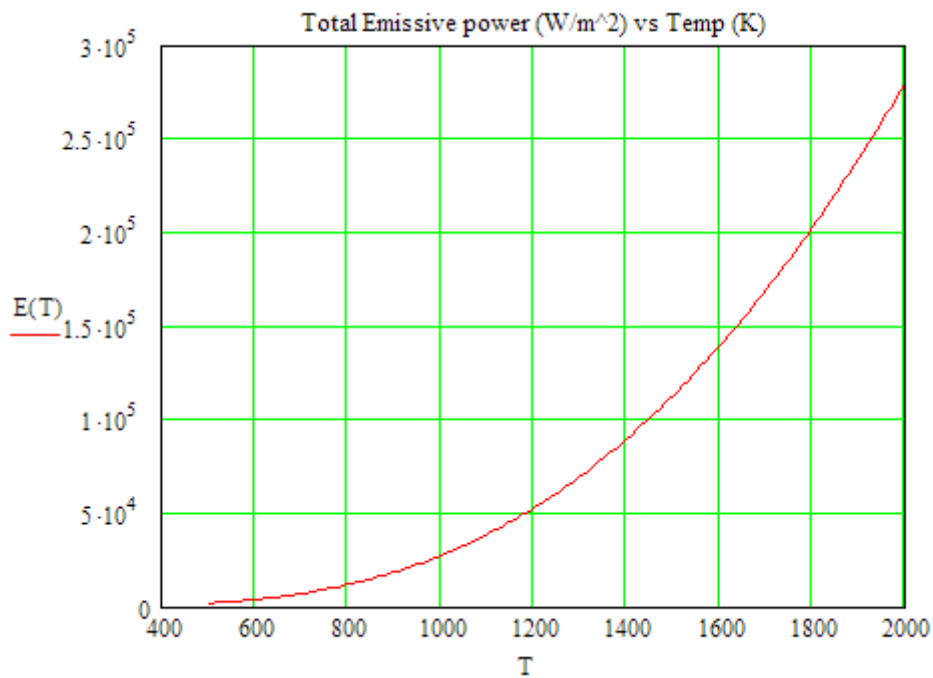
Prob.5A.9. Spectral hemispherical emissivity (monochromatic emissivity) of fire brick at 750 K as a function of wave length is as follows:

$\varepsilon 1 = 0.1$ for $\lambda = 0$ to 2 μm, $\varepsilon 2 = 0.6$ for $\lambda = 2$ to 14 μm, and $\varepsilon 3 = 0.3$ for $\lambda = 14$ to ∞.Calculate total hemispherical emissivity ε, over all wave lengths. [VTU – July–Aug. 2004]

Mathcad Solution:

This problem is similar to the previous problem.

So, we will enter the data, and use the Mathcad Functions already written:

Data:

$T := 750$ K....temperature

$\varepsilon 1 := 0.1$ ...emissiviy in wavelength range: 0 to 2 μm

$\varepsilon 2 := 0.6$ ...emissiviy in wavelength range: 2 to 14 μm

$\varepsilon 3 := 0.3$ ...emissiviy in wavelength range: 14 μm to ∞.

$\sigma := 5.67 \cdot 10^{-8}$ W/(m^2.K^4)....Stefan-Boltzmann const.

$\lambda_1 := 2$ μm

$\lambda_2 := 14$ μm

$\lambda_3 := \infty$

Results:

$F_{0\_\lambda 1}(T) := F_{0\_\lambda}(M1, \lambda_1 \cdot T)$ i.e: $F_{0\_\lambda 1}(T) = 0.01375$

$F_{0\_\lambda 2}(T) := F_{0\_\lambda}(M1, \lambda_2 \cdot T)$ i.e.: $F_{0\_\lambda 2}(T) = 0.9237$

$F_{0\_\lambda 3} := F_{0\_\lambda}(M1, 100001)$ ....Note: For λT = infinity, we have: $F_{0\_\lambda 3} = 1$

$\varepsilon(T) := \varepsilon 1 \cdot F_{0\_\lambda 1}(T) + \varepsilon 2 \cdot (F_{0\_\lambda 2}(T) - F_{0\_\lambda 1}(T)) + \varepsilon 3 \cdot (F_{0\_\lambda 3} - F_{0\_\lambda 2}(T))$

...total hemisph. emissivity

i.e.: $\varepsilon(T) = 0.57$ **..total hemisph. emissivity ... Ans.**

$E(T) := \varepsilon(T) \cdot \sigma \cdot T^4$ W/m^2....total emissive power

i.e.: $E(T) = 1.023 \times 10^4$ **W/m^2....total emissive power.... Ans.**

Plot the variation of total emissivity ε and total emissive power E as Temp T varies from 500 K to 2000 K:

$$T := 500, 510 .. 2000 \qquad T := 500, 510 .. 2000$$

T =	ε(T) =	E(T) =
500	0.542	$1.922 \cdot 10^3$
600	0.56	$4.117 \cdot 10^3$
700	0.569	$7.747 \cdot 10^3$
800	0.571	$1.325 \cdot 10^4$
900	0.566	$2.105 \cdot 10^4$
1000	0.555	$3.15 \cdot 10^4$
1100	0.541	$4.489 \cdot 10^4$
1200	0.523	$6.148 \cdot 10^4$
1300	0.503	$8.143 \cdot 10^4$
1400	0.481	$1.049 \cdot 10^5$
1500	0.459	$1.319 \cdot 10^5$
1600	0.438	$1.626 \cdot 10^5$
1700	0.416	$1.972 \cdot 10^5$
1800	0.396	$2.356 \cdot 10^5$
1900	0.376	$2.78 \cdot 10^5$
2000	0.358	$3.246 \cdot 10^5$

Total hemisph. emissivity vs Temp (K)

Download free eBooks at bookboon.com

Total Emissive power (W/m^2) vs Temp (K)

===

Prob.5A.10. For a black body maintained at 390 K, determine:

1) total emissive power
2) wave length at which max. monochromatic emissive power occurs
3) Max. monochromatic emissive power [M.U. May 1999]

Mathcad Solution:

Data:

$C1 := 3.742 \cdot 10^8$ W.μm^4/m^2 $C2 := 1.4387 \cdot 10^4$ μm.K

$T := 390$ K $\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K^4... Stefan Boltzmann constant.

Calculations:

$$E_b := \sigma \cdot T^4$$

i.e. $E_b = 1.312 \times 10^3$ W/m^2....Total emissive power.... Ans.

We have:

$$\lambda_m := \frac{2897.8}{T}$$

i.e. $\lambda_m = 7.43$ microns....wave length for max. mono. emissive power at 390 K.... Ans.

And:

$$E_{b\lambda} := \frac{C1 \cdot \lambda_m^{-5}}{\exp\left(\frac{C2}{\lambda_m \cdot T}\right) - 1}$$

i.e. $E_{b\lambda} = 116.129$ W/m^2.micron...Max. monochr. emissive power...Ans.

===

"**Prob.5A.11.** For a body, of area 0.15 m^2, at an effective temp of 900 K, calculate the following: (i) Total rate of energy emission (E) (ii) Intensity of normal radiation (I_{bn}), and (iii) Wave length of max. monochromatic emissive power".

Also plot the variation of λ_m with T , varying T from 500 K to 2000 K.

"EES Solution:"

"Data:"

A = 0.15 [m^2]

T = 900 [K]

sigma = 5.67E-08 [W/m^2-K^4] "…Stefan Boltzmann constant"

"Calculations:"

E_b = sigma * T^4 "[W/m^2]…Total Emissive power"

E = E_b * A "[W]…Total rate of energy emission"

I_bn = E_b / pi "[W/m^2]…Intensity of normal radiation"

lambda_m * T = 2898 "…by Wein's displacement Law.. finds lambda_max, microns"

Results:

Unit Settings: SI C Pa J mass deg

A = 0.15 [m²]	E = 5580 [W]	E_b = 37201 [W/m²]
I_{bn} = 11841 [W/m²]	λ_m = 3.22 [microns]	σ = 5.670E-08 [W/m²K⁴]
T = 900 [K]		

Thus:

Total rate of energy emission = E = 5580 W

Intensity of normal radiation = 11841 W/m^2

Wave length of max. monochromatic emissive power = 3.22 microns …Ans.

Also plot the variation of λ_m with T, varying T from 500 K to 2000 K:

First, compute the Parametric Table:

	T [K]	λ_m [microns]	E [W]	I_{bn} [W/m2]
Run 1	500	5.796	531.6	1128
Run 2	600	4.83	1102	2339
Run 3	700	4.14	2042	4333
Run 4	800	3.623	3484	7393
Run 5	900	3.22	5580	11841
Run 6	1000	2.898	8505	18048
Run 7	1100	2.635	12452	26424
Run 8	1200	2.415	17636	37425
Run 9	1300	2.229	24291	51547
Run 10	1400	2.07	32673	69334
Run 11	1500	1.932	43057	91369
Run 12	1600	1.811	55738	118280
Run 13	1700	1.705	71035	150740
Run 14	1800	1.61	89282	189462
Run 15	1900	1.525	110838	235206
Run 16	2000	1.449	136080	288771

Now, plot the results:

==

"**Prob.5A.12.** Considering an Industrial furnace at 2500 K as a black body, find out the following: (i) monochromatic emissive power at a wave length of 1.2 microns, (ii) wave length at which emission is maximum, (iii) max. emissive power, and (iv) total emissive power".

"EES Solution:"

"Data:"

T = 2500 [K]

lambda = 1.2 [microns]

sigma = 5.67E-08 [W/m^2-K^4] "…Stefan Boltzmann constant"

C1 = 3.742E08 [W-micron^4/m^2] "…const. in Planck's law eqn"

C2 = 1.4388E04 [m-K] "…const. in Planck's law eqn"

"Calculations:"

E_lambda_b = C1 * lambda^(-5) / (exp(C2 / (lambda * T)) -1) "[W/m^2]…monochr. emissive power at lambda = 2 microns"

lambda_m * T = 2898 "…by Weins displacement Law.. finds lambda_max, microns"

E_lamda_b_max = 1.285E-05 * T^5 "[W/m^2 per m length]…Max. mono. emissive power"

E_b = sigma * T^4 "[W/m^2]…Total emissive power"

Results:

Unit Settings: SI C kPa kJ mass deg

C1 = 3.742E+08 [W-micron^4/m^2]

C2 = 14388 [m-K]

E_b = 2.215E+06 [W/m^2]

$E_{\lambda,b}$ = 1.253E+06 [W/m^2]

$E_{lamda,b,max}$ = 1.255E+12 [W/m$^2 \cdot$ m·length]

λ = 1.2 [microns]

λ_m = 1.159 [microns]

σ = 5.670E-08 [W/m^2·K^4]

T = 2500 [K]

Thus:

Monochr. emissive power at 1.2 μm = E_lambda_b = 1.253E06 W/m^2

Wave length at which emission is max = λ_m = 1.159 μm

Max. mono. emissive power = E_lambda_b_max = 1.255E12 W/m^2 per m length

Total emissive power = E_b = 2.215E06 W/m^2 … Ans.

==

Prob.5A.13: Draw a plot of Planck's Law for spectral distribution with EXCEL:

EXCEL Solution:

We have already stated the Planck's Law, i.e.

$$E_{b\lambda}(\lambda, T) := \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1}$$ ….$E_{b\lambda}$ as a function of λ and T

Where C1 and C2 are constants, given as:

C1 = 3.742E08 [W-micron^4/m^2]"…const. in Planck's law eqn"

C2 = 1.4388E04 [m-K]"…const. in Planck's law eqn"

Now, let us write a VBA Function in EXCEL to produce a Table of λ against T and plot the same for T = 5800 K, 1000 K, 500 K and 100 K:

Following are the steps in EXCEL:

1. In EXCEL go to: Developer – Visual Basic:
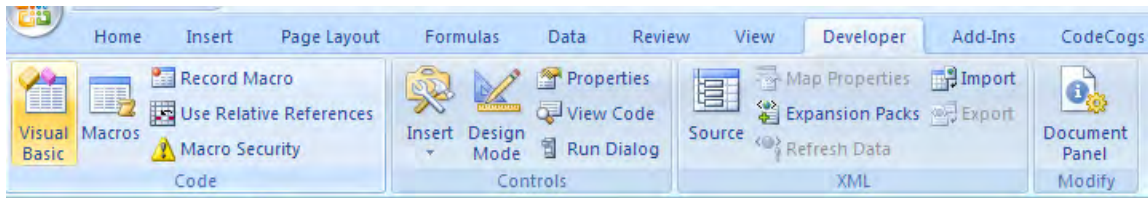
2. Click on Visual Basic – Insert:
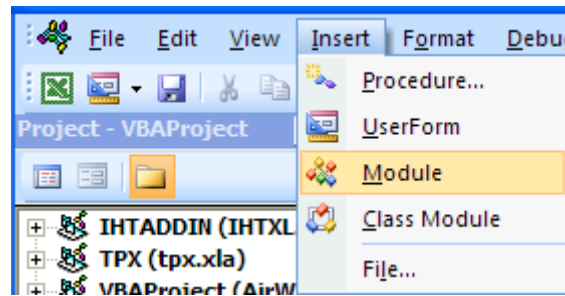
3. Click on Module and a blank window (Module 1) presents itself. Write the code in that window as shown below:

```
(General)                                                    Plancks_Law

    Option Explicit
    Function Plancks_Law(Lambda As Double, T As Double) As Double

    'Planck's Law for Spectral distribution: Ref: p.645, HMT by MT
    'Finds E_b_lambda for given Lambda and T

    Dim C_1 As Double, C_2 As Double, AA As Double, BB As Double

    C_1 = 374200000# 'W.micron^4/m2
    C_2 = 14387#     'micron.K
    AA = C_1 * Lambda ^ (-5)
    BB = Exp(C_2 / (Lambda * T)) - 1
    Plancks_Law = AA / BB 'W/micron.K

    End Function
```

4. Now, this Function is available in EXCEL worksheet like any other built-in Function of EXCEL, under the 'User Defined' category. Let us use it to produce the Table and plot as shown below:

Only part of the Table is shown below. Note the formula used (i.e. the VBA Function written above) in the Formula bar:

	C10		f_x	=Plancks_Law(B10,5800)		
	A	**B**	**C**	**D**	**E**	**F**
4						
5		**Planck's Law for Spectral distribution: (Ref: MT, p.645)**				
6						
7			T = 5800 K	T = 1000 K	T = 500 K	T = 100 K
8		Lambda (microns)	E_b_lambda (W/m^2/micron)			
9						
10		0.06	0.000534275			
11		0.07	0.090776083			
12		0.08	3.905862682			
13		0.09	67.94688745			
14		0.1	631.4722659			
15		0.2	4803754.872			
16		0.3	39508670.79			
17		0.4	74216005.51	8.7562E-06		
18		0.5	84480620.57	0.00381825		
19		0.6	78325115.48	0.18564775		
20		0.7	66281071.45	2.64010463		
21		0.8	53835149.43	17.6770775		
22		0.9	42996046.38	72.3523627		
23		1	34181499.37	211.304778	0.0001193	
24		1.2	21788716.23	934.05258	0.0058015	
25		1.4	14253369.15	2395.97531	0.0825033	
26		1.6	9611245.643	4440.54435	0.5524087	

◢	A	B	C	D	E	F
145		740	0.000502235	8.5897E-05	4.253E-05	7.8579E-06
146		760	0.000451438	7.7226E-05	3.825E-05	7.0815E-06
147		780	0.000406904	6.9622E-05	3.449E-05	6.3986E-06
148		800	0.000367729	6.2931E-05	3.118E-05	5.7961E-06
149		820	0.000333157	5.7025E-05	2.826E-05	5.2629E-06
150		840	0.000302554	5.1795E-05	2.568E-05	4.7895E-06
151		860	0.000275386	4.7152E-05	2.338E-05	4.3682E-06
152		880	0.000251199	4.3018E-05	2.133E-05	3.9923E-06
153		900	0.000229611	3.9327E-05	1.951E-05	3.6559E-06
154		920	0.000210293	3.6023E-05	1.787E-05	3.3541E-06
155		940	0.000192964	3.3059E-05	1.64E-05	3.0829E-06
156		960	0.000177384	3.0394E-05	1.508E-05	2.8386E-06
157		980	0.000163346	2.7992E-05	1.389E-05	2.6179E-06
158		1000	0.000150669	2.5823E-05	1.282E-05	2.4183E-06

Now, plot the results:

==
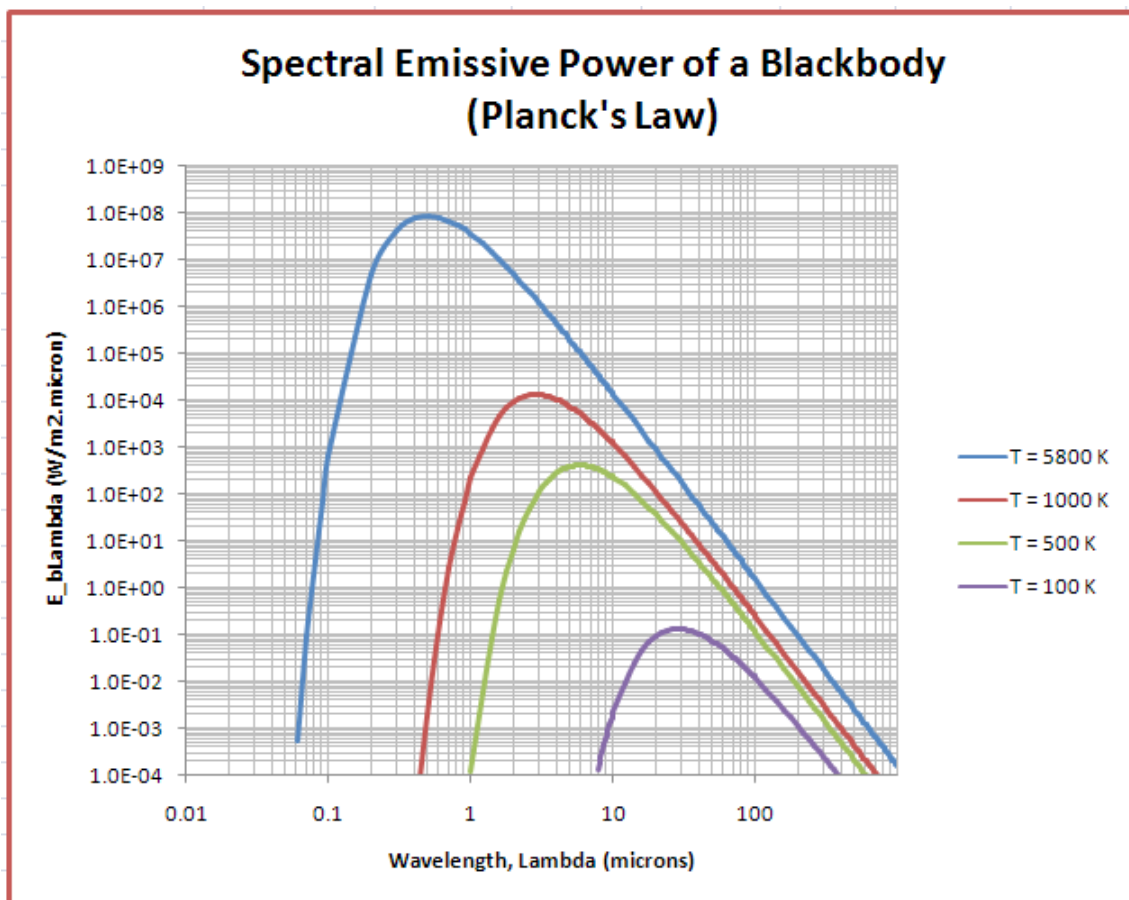
Prob.5A.14: Write an EXCEL VBA program to generate the Radiation Functions (i.e. radiation emitted in a wave band as a fraction of Total emissive power) against λT.

EXCEL Solution:

Recollect Planck's Law:

$$E_{b\lambda}(\lambda) = \frac{C_1 \cdot \lambda^{-5}}{\exp\left(\dfrac{C_2}{\lambda \cdot T}\right) - 1} \qquad W/(m^2.\mu m).....(13.7)$$

where, $\qquad C_1 = 3.742 \cdot 10^8 \qquad W.\mu m^4/m^2$

and, $\qquad C_2 = 1.4387 \cdot 10^4 \qquad \mu m.K$

Radiation from a wave band:

Often, it is required to know the amount of radiation emitted in a given wave band, i.e. in a wave length interval between λ_1 and λ_2. This is expressed as a fraction of the total emissive power and is written as

$F\lambda_{1\text{-}}\lambda_2$. Then, we can write:

$$F_{\lambda 1\_\lambda 2} = \frac{\int_{\lambda 1}^{\lambda 2} E_{b\lambda}\, d\lambda}{\int_{0}^{\infty} E_{b\lambda}\, d\lambda} = \frac{1}{\sigma \cdot T^4} \cdot \int_{\lambda 1}^{\lambda 2} E_{b\lambda}\, d\lambda$$

i.e.

$$F_{\lambda 1\_\lambda 2} = \frac{1}{\sigma \cdot T^4} \cdot \left(\int_{0}^{\lambda 2} E_{b\lambda}\, d\lambda - \int_{0}^{\lambda 1} E_{b\lambda}\, d\lambda \right)$$

i.e.   ......(13.15)

$$F_{\lambda 1\_\lambda 2} = F_{0\_\lambda 2} - F_{0\_\lambda 1}$$

Above formula is not very convenient to use, since $E_b\lambda$ depends on absolute temperature T, and it is not practicable to tabulate $F_{0\text{-}}\lambda$ for each T. This difficulty is overcome by expressing $F_{0\text{-}}\lambda$ as follows:

$$F_{0\_\lambda} = \frac{\int_{0}^{\lambda} E_{b\lambda}\, d\lambda}{\sigma \cdot T^4} = \frac{\int_{0}^{\lambda \cdot T} E_{b\lambda}\, d(\lambda \cdot T)}{\sigma \cdot T^5}$$

i.e.   ....(13.16)

$$F_{0\_\lambda} = f(\lambda \cdot T)$$

So, we shall, first, write a VBA Function to calculate $F_{0\text{-}\lambda}$ and then, use it to write another Function for $F_{\lambda 1\text{-}\lambda 2}$.

Following are the VBA Functions, where *integrals are calculated by numerical method*, using the Trapezoidal rule.

```
(General)                                               ▼   F_zero_lambdaT1_lambdaT2_alternative

    Option Explicit
    Function F(x As Double) As Double

     'This function is used to calculate the fraction F_0_lambda, Ref: Incropera

     Dim C_1 As Double, C_2 As Double, sigma As Double

     C_1 = 374200000#  'W.micron^4/m2
     C_2 = 14388#      'micron.K
     sigma = 0.0000000567 'W/m2.K^4
      F = C_1 / (sigma * x ^ 5 * (Exp(C_2 / x) - 1))
    End Function
    Function F_zero_lambdaT_Low_lambdaT_High_trapezoidal(LambdaT_Low As Double, _
    LambdaT_High As Double, n As Integer) As Double
    'Gives the fraction F_zero_Lambda by integration using Trapezoidal rule

     'Input: LambdaT_Low, LambdaT_High in micron.K, n is the no. of equal intervals

     Dim dx As Double, sum As Double, y As Double, i As Integer

     'If LambdaT_Low is less than 21, F(x) goes to extremely low value and gives error!
     If LambdaT_Low < 21 Then
         LambdaT_Low = 21
     End If
     If LambdaT_High > 100000 Then
         LambdaT_High = 100000
     End If
     dx = (LambdaT_High - LambdaT_Low) / n
     sum = (F(LambdaT_Low) + F(LambdaT_High)) / 2
     For i = 1 To (n - 1)
         y = F(LambdaT_Low + (dx * i))
         sum = sum + y
     Next i
     sum = sum * dx
     F_zero_lambdaT_Low_lambdaT_High_trapezoidal = sum
    End Function
```

Now, we shall use these Functions to generate a Table of F0-λT against λT as shown below. Note that we have taken lower value of λT as 20 micron.K for reasons stated in the program. In addition, we have compared the thus calculated values to the Table values given in Incropera.

Also, note that 1000 steps have been taken while implementing the Trapezoidal rule. Errors as compared to Table values in Incropera are within 1.08%.

| | E7 | ▼ | f_x | =F_zero_lambdaT_Low_lambdaT_High_trapezoidal($C$7,D7,1000) | | |

	B	C	D	E	F	G
1						
2						
3		**Band Emission: Fraction of radiation emitted in a wave length band: F_0_Lambda**				
4		Ref: Incropera				
5						
6		LambdaT_Low (micron.K)	LambdaT_High (micron.K)	F_zero_Lambda1_Lambda2	Incropera_Value	Error (%)
7		20	200	4.77406E-27		
8			400	2.02204E-12		
9			600	9.62015E-08		
10			800	1.67442E-05		
11			1000	0.000324456	0.000321	-1.076699448
12			1200	0.002150487		
13			1400	0.00783215		
14			1600	0.019796645		
15			1800	0.039459178		
16			2000	0.066884133		
17			2200	0.10107475		
18			2400	0.140465168		
19			2600	0.183343203		
20			2800	0.228120007		
21			2898	0.250338338	0.250108	-0.092095456
22			3000	0.273461769		

| | G37 | ▼ | f_x | =(F37-E37)*100/F37 | | |

	B	C	D	E	F	G
35			5600	0.701155044		
36			5800	0.720259702		
37			6000	0.737912358	0.737818	-0.012788728
38			6200	0.754228527		
39			6400	0.769316337		
40			6600	0.783276281		
41			6800	0.796201259		
42			7000	0.808176805		
43			7200	0.819281432		
44			7400	0.829587045		
45			7600	0.839159389		
46			7800	0.848058508		
47			8000	0.856339194		
48			8500	0.874652534		
49			9000	0.890069405		
50			9500	0.903121203		
51			10000	0.914231511	0.914199	-0.003556189

	E69		fx	=F_zero_lambdaT_Low_lambdaT_High_trapezoidal($C$7,D69,1000)		
	B	C	D	E	F	G
62			19000	0.983468974		
63			20000	0.985616976	0.985602	-0.00151948
64			25000	0.992228641		
65			30000	0.995353822		
66			40000	0.997980671		
67			50000	0.998966543		
68			75000	0.999725719	0.999713	-0.001272301
69			100000	0.999917921	0.999905	-0.001292269

Now, plot these results in EXCEL:

Fraction of total blackbody emission in the spectral band (0 - λ) as a function of λT

==
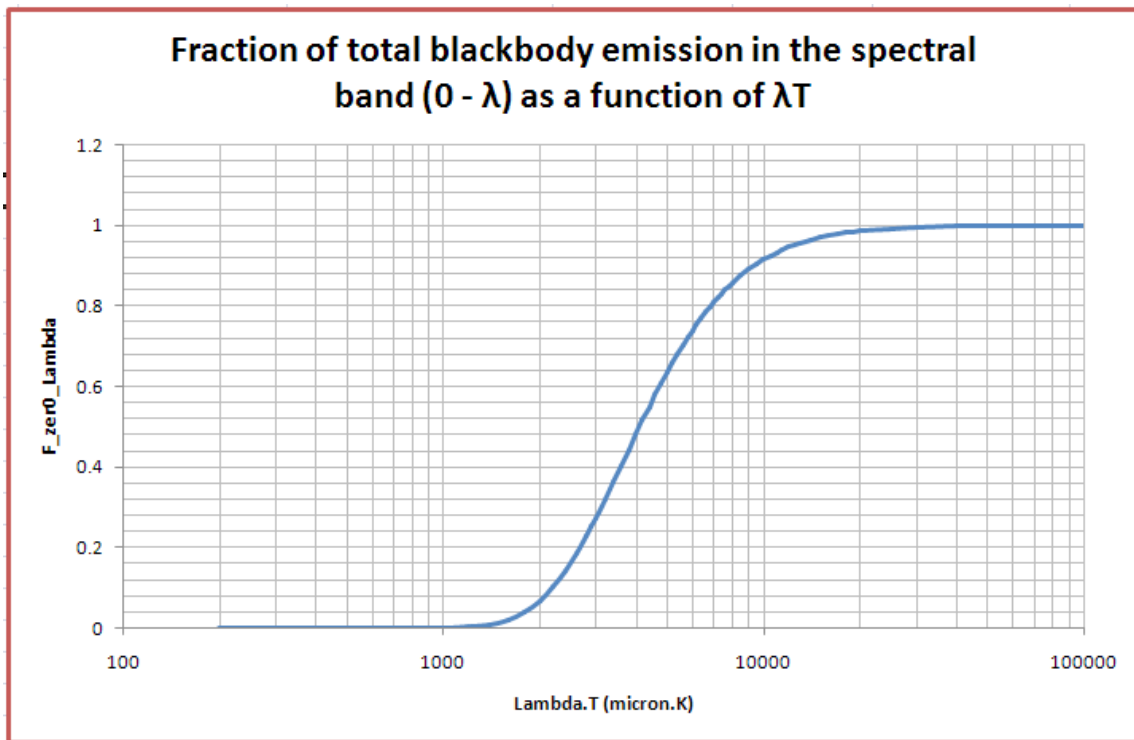
Prob.5A.15. Spectral hemispherical emissivity (monochromatic emissivity) of fire brick at 750 K as a function of wave length is as follows:

ε1 = 0.1 for λ = 0 to 2 μm, ε2 = 0.6 for λ = 2 to 14 μm, and ε3 = 0.3 for λ = 14 to ∞. Calculate total hemispherical emissivity ε, over all wave lengths. [VTU – July–Aug. 2004]

Also, plot the variation of total emissivity ε and total emissive power E as Temp T varies from 500 K to 2000 K:

Note: This is the same as Prob.5A.9 which was solved with Mathcad.

EXCEL Solution:

Let us use the VBA Function written above for F0-λ while solving this problem.

1. Set up the EXCEL worksheet, enter data and name the cells, as shown below:

		lambda2			f_x 14	
	A	B	C		D	E
96						
97						
98			Prob.5A.15.			
99						
100			Data:			
101			T		750	K
102			epsilon1		0.1	emissivity (0 - 2μ)
103			epsilon2		0.6	emissivity (2 - 14μ)
104			epsilon3		0.3	emissivity (14μ - infinity)
105			sigma		5.67E-08	W/m^2.K^4
106			lambda1		2	microns
107			lambda2		14	microns
108			lambda3		1.00E+10	microns

2. Do the calculations:

		F_0_lambdaT_1			f_x =F_zero_lambdaT_Low_lambdaT_High_trapezoidal(0,lambdaT_1,1000)	
	A	B	C	D	E	F
109						
110			Calculations:			
111						
112			lambdaT_1	1500	micron.K	multiplying lambda and T
113			lambdaT_2	10500	micron.K	
114			lambdaT_3	7.5E+12	micron.K	
115						
116			Now, get F_0_lambdaT using the VBA Function:			
117						
118			F_0_lambdaT_1	0.012849488		Using VBA Functions for F_0_lambdaT
119			F_0_lambdaT_2	0.923721652		
120			F_0_lambdaT_3	0.999917917		
121						

Note: In the above screen shot, see the VBA Function invoked in cell D118, in the Formula bar.

3. Continuing with the calculation:

	epsilon	▾	f_x	=epsilon1*F_0_lambdaT_1+epsilon2*(F_0_lambdaT_2-F_0_lambdaT_1)+epsilon3*(F_0_lambdaT_3-F_0_lambdaT_2)			
	A	B	C	D	E	F	G	H
121								
122			Therefore:		$\varepsilon(T) := \varepsilon1 \cdot F_{0\_\lambda 1}(T) + \varepsilon2 \cdot \left(F_{0\_\lambda 2}(T) - F_{0\_\lambda 1}(T) \right) + \varepsilon3 \cdot \left(F_{0\_\lambda 3} - F_{0\_\lambda 2}(T) \right)$			
123			**Total hemisph. Emissivity:**			...total hemisph. emissivity		
124			epsilon	0.570667127	....Ans.	$E(T) := \varepsilon(T) \cdot \sigma \cdot T^4$	W/m²....total emissive power	
125								
126			**Total Emissive Power: E**	10237.902	W/m^2 ... Ans.			

Thus:

Total hemisph. Emissivity = e = 0.571 … Ans.

Total Emissive Power = E = 10237.9 W /m^2 … Ans.

To plot the variation of total emissivity ε and total emissive power E as Temp T varies from 500 K to 2000 K:

1. Set up a Table as shown, and enter the first row below the column titles, i.e. row 133, using the Formulas already written. However, take care to enter T with *relative reference* in the formulas, so that we can drag-copy the first row to fill up the entire Table:

| | G133 | | f_x | =F_zero_lambdaT_Low_lambdaT_High_trapezoidal(0,D133,1000) | | | | | | |

	A	B	C	D	E	F	G	H	I	J	K
132			T (K)	lambdaT_1(micron.K)	lambdaT_2(micron.K)	lambdaT_3(micron.K)	F_0_lambdaT_1	F_0_lambdaT_2	F_0_lambdaT_3	epsilon	E (W/m^2)
133			500	1000	7000	5E+12	0.000320733	0.80811908	0.999917917	0.54225	1921.601034
134			550								
135			600								
136			650								
137			700								
138			750								
139			800								
140			850								
141			900								
142			950								
143			1000								
144			1050								
145			1100								
146			1150								
147			1200								
148			1250								
149			1300								
150			1350								
151			1400								
152			1450								
153			1500								
154			1550								
155			1600								
156			1650								
157			1700								
158			1750								
159			1800								
160			1850								
161			1900								
162			1950								
163			2000								

Note how the VBA Function is entered in cell G133, in the Formula bar. Similarly, for all other columns, use relative references to appropriate quantities.

2. Now, select row 133 i.e. from cell D133 to cell K133, and drag-copy to the end of the Table, i.e. up to cell K163. Immediately, all calculations are made and the Table is filled up:

|	B	C	D	E	F	G	H	I	J	K
132		T (K)	lambdaT_1(micron.K)	lambdaT_2(micron.K)	lambdaT_3(micron.K)	F_0_lambdaT_1	F_0_lambdaT_2	F_0_lambdaT_3	epsilon	E (W/m^2)
133		500	1000	7000	5E+12	0.000320733	0.80811908	0.999917917	0.542	1921.601
134		550	1100	7700	5.5E+12	0.000911145	0.843645005	0.999917917	0.553	2867.181
135		600	1200	8400	6E+12	0.002134036	0.871206025	0.999917917	0.560	4117.044
136		650	1300	9100	6.5E+12	0.004316168	0.892826594	0.999917917	0.566	5725.270
137		700	1400	9800	7E+12	0.007789953	0.909975128	0.999917917	0.569	7747.171
138		750	1500	10500	7.5E+12	0.012849488	0.923721652	0.999917917	0.571	10237.902
139		800	1600	11200	8E+12	0.019718434	0.934852175	0.999917917	0.571	13251.142
140		850	1700	11900	8.5E+12	0.028533364	0.94394967	0.999917917	0.569	16837.973
141		900	1800	12600	9E+12	0.039341265	0.951450973	0.999917917	0.566	21046.021
142		950	1900	13300	9.5E+12	0.052107386	0.957686825	0.999917917	0.561	25918.902
143		1000	2000	14000	1E+13	0.066729199	0.962910186	0.999917917	0.555	31495.933
144		1050	2100	14700	1.05E+13	0.083052858	0.967316428	0.999917917	0.549	37812.099
145		1100	2200	15400	1.1E+13	0.10088958	0.971057862	0.999917917	0.541	44898.205
146		1150	2300	16100	1.15E+13	0.120030358	0.974254285	0.999917917	0.532	52781.176
147		1200	2400	16800	1.2E+13	0.140258237	0.977000715	0.999917917	0.523	61484.448
148		1250	2500	17500	1.25E+13	0.161357934	0.979373116	0.999917917	0.513	71028.425
149		1300	2600	18200	1.3E+13	0.18312292	0.981432674	0.999917917	0.503	81430.949
150		1350	2700	18900	1.35E+13	0.20536028	0.983229021	0.999917917	0.492	92707.775
151		1400	2800	19600	1.4E+13	0.227893746	0.984802687	0.999917917	0.481	104873.028
152		1450	2900	20300	1.45E+13	0.250565263	0.986186975	0.999917917	0.471	117939.633
153		1500	3000	21000	1.5E+13	0.273235479	0.987409416	0.999917917	0.460	131919.699
154		1550	3100	21700	1.55E+13	0.295783431	0.988492894	0.999917917	0.449	146824.876
155		1600	3200	22400	1.6E+13	0.318105697	0.989456534	0.999917917	0.438	162666.663
156		1650	3300	23100	1.65E+13	0.340115205	0.990316401	0.999917917	0.427	179456.677
157		1700	3400	23800	1.7E+13	0.361739836	0.99108605	0.999917917	0.416	197206.888
158		1750	3500	24500	1.75E+13	0.382920952	0.991776974	0.999917917	0.406	215929.819
159		1800	3600	25200	1.8E+13	0.40361191	0.992398958	0.999917917	0.396	235638.708
160		1850	3700	25900	1.85E+13	0.423776624	0.992960368	0.999917917	0.386	256347.652
161		1900	3800	26600	1.9E+13	0.443388211	0.993468383	0.999917917	0.376	278071.719
162		1950	3900	27300	1.95E+13	0.462427733	0.993929189	0.999917917	0.367	300827.043
163		2000	4000	28000	2E+13	0.480883053	0.994348131	0.999917917	0.358	324630.895

3. Now, plot the results in EXCEL:
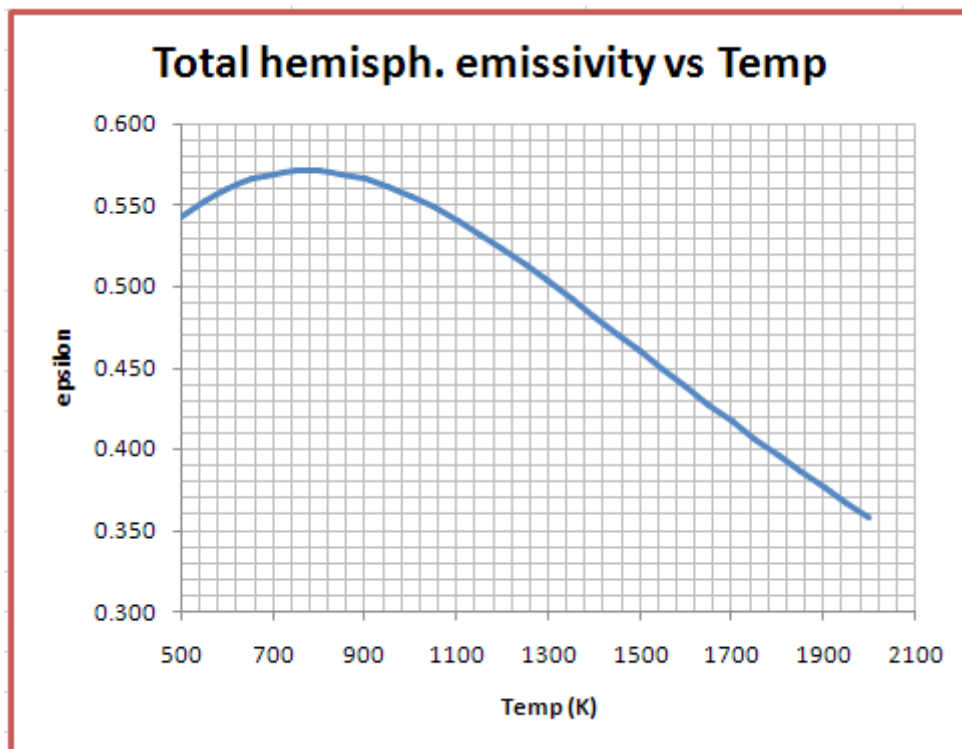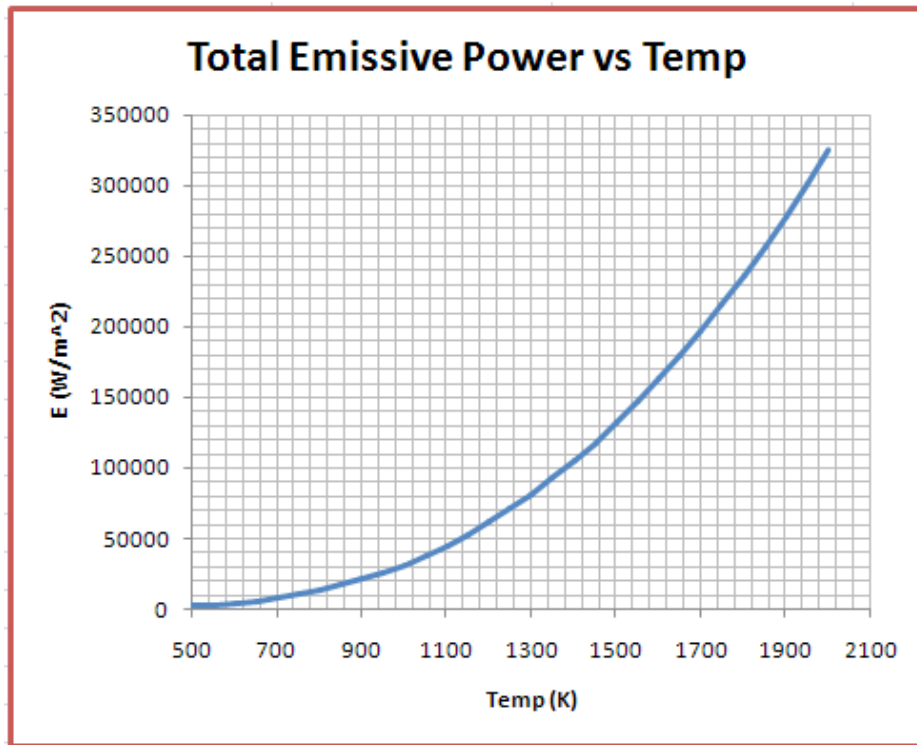
Note that results obtained with EXCEL match very well with those obtained with Mathcad, as they should.

===

5B View factors (or, Shape factor, configuration factor) and Radiation energy exchange between Black bodies:

Recollect:

Definition of view factor: it is the fraction of energy emitted by surface 1, which is intercepted directly by surface 2.

Note: Often, while solving radiation problems, determination of the view factor is the most difficult part. It will be *useful to keep in mind* the definition of view factor, summation rule, reciprocity relation, superposition rule and symmetry rule while attempting to find out the view factors.

Radiation energy exchange between black bodies:

Analysis of heat exchange between two black bodies is simpler since a black body absorbs all the radiation impinging on it and none is reflected.

Consider two black surfaces A_1 and A_2 at temperatures T1 and T2, exchanging radiation energy with each other.

Then:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad W....(13.40)$$

Prob.5B.1 Draw the plots for View factors of:
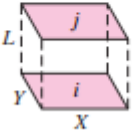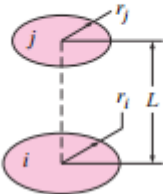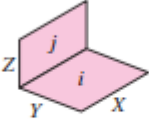
1) **Aligned, parallel rectangles of equal size**
2) **Coaxial, parallel disks, and**
3) **Perpendicular rectangles with a common edge.**

View factor relationships for a few important shapes, given at the beginning of this chapter, are reproduced below. (Ref: Cengel).

Now, we shall write Mathcad Functions for View factors, so that we need not refer to the graphs while doing calculations.

We will also use these Functions to draw the graphs.

View factor expressions for some common geometries of finite size (3D)

Geometry	Relation
Aligned parallel rectangles	$\overline{X} = X/L$ and $\overline{Y} = Y/L$ $F_{i \to j} = \dfrac{2}{\pi \overline{X} \overline{Y}} \left\{ \ln\left[\dfrac{(1 + \overline{X}^2)(1 + \overline{Y}^2)}{1 + \overline{X}^2 + \overline{Y}^2} \right]^{1/2} \right.$ $+ \overline{X}(1 + \overline{Y}^2)^{1/2} \tan^{-1} \dfrac{\overline{X}}{(1 + \overline{Y}^2)^{1/2}}$ $+ \overline{Y}(1 + \overline{X}^2)^{1/2} \tan^{-1} \dfrac{\overline{Y}}{(1 + \overline{X}^2)^{1/2}}$ $\left. - \overline{X} \tan^{-1} \overline{X} - \overline{Y} \tan^{-1} \overline{Y} \right\}$
Coaxial parallel disks	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \dfrac{1 + R_j^2}{R_i^2}$ $F_{i \to j} = \dfrac{1}{2} \left\{ S - \left[S^2 - 4 \left(\dfrac{r_j}{r_i} \right)^2 \right]^{1/2} \right\}$
Perpendicular rectangles with a common edge	$H = Z/X$ and $W = Y/X$ $F_{i \to j} = \dfrac{1}{\pi W} \left(W \tan^{-1} \dfrac{1}{W} + H \tan^{-1} \dfrac{1}{H} \right.$ $- (H^2 + W^2)^{1/2} \tan^{-1} \dfrac{1}{(H^2 + W^2)^{1/2}}$ $+ \dfrac{1}{4} \ln \left\{ \dfrac{(1 + W^2)(1 + H^2)}{1 + W^2 + H^2} \right.$ $\times \left[\dfrac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{W^2}$ $\left. \left. \times \left[\dfrac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{H^2} \right\} \right)$

----

1. **View factors for aligned, parallel rectangles: (See the fig. above for nomenclature)**

We define:

$$XX = \frac{X}{L} \qquad YY = \frac{Y}{L}$$

$$A(XX, YY) := \frac{2}{\pi \cdot XX \cdot YY} \qquad B(XX, YY) := \ln\left[\sqrt{\frac{\left(1 + XX^2\right) \cdot \left(1 + YY^2\right)}{1 + XX^2 + YY^2}} \right]$$

$$C(XX, YY) := XX \cdot \left(1 + YY^2\right)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{XX}{\left(1 + YY^2\right)^{\frac{1}{2}}}\right]$$

$$D(XX, YY) := YY \cdot \left(1 + XX^2\right)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{YY}{\left(1 + XX^2\right)^{\frac{1}{2}}}\right]$$

$$E(XX) := XX \cdot \operatorname{atan}(XX) \qquad F(YY) := YY \cdot \operatorname{atan}(YY)$$

$$F_{ij}(XX, YY) := A(XX, YY) \cdot (B(XX, YY) + C(XX, YY) + D(XX, YY) - E(XX) - F(YY))$$

....View factor for aligned parallel rectangles

Now, to plot the graphs:

$$XX := 0.1, 0.2 .. 30 \quad ...\text{define a range variable:}$$

View factor for parallel rectangles

Y/L = 0.1
Y/L = 0.2
Y/L = 0.4
Y/L = 0.6
Y/L = 1.0
Y/L = 2.0
Y/L = 4.0
Y/L = 10.0

===
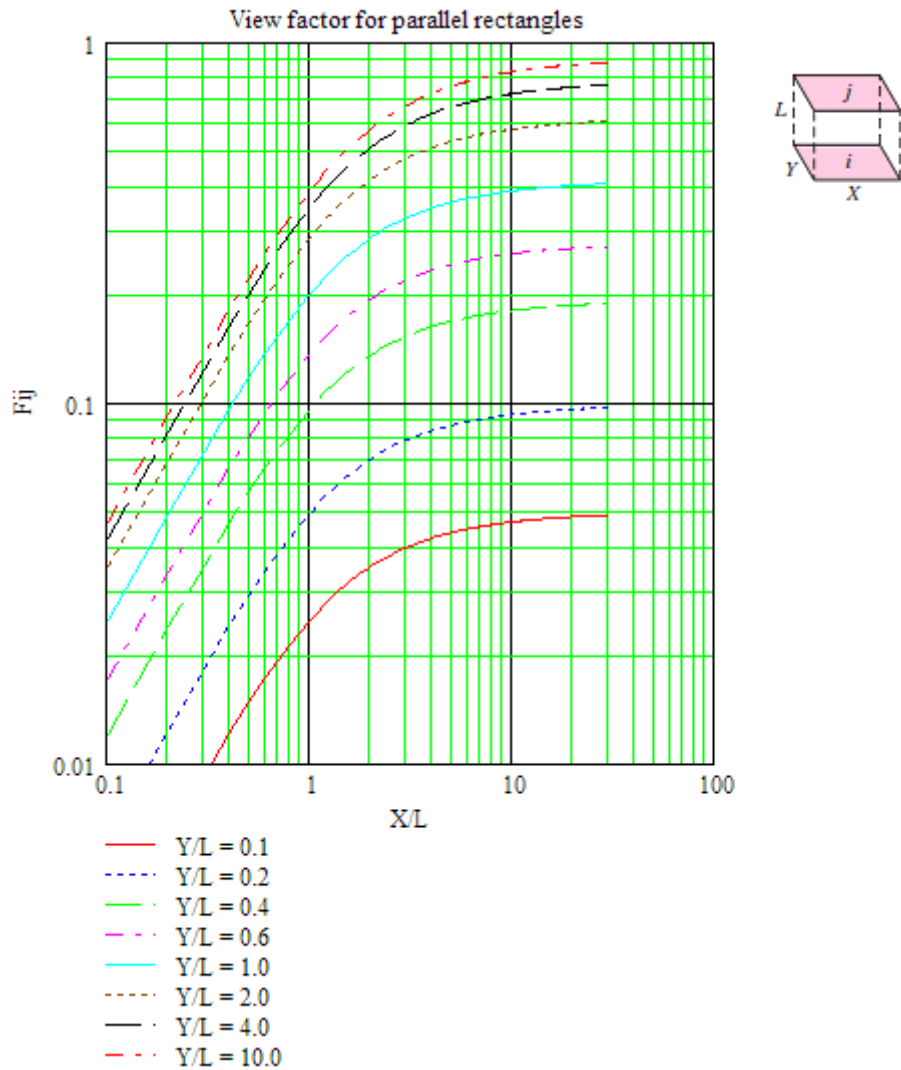
We write the Mathcad Function for F12 for aligned, parallel rectangles as follows:

Here,

Inputs are: X, Y and L.

Output is: F12.

$F12\_parallel\_rectangles(X, Y, L) :=$

$$XX \leftarrow \frac{X}{L}$$

$$YY \leftarrow \frac{Y}{L}$$

$$A \leftarrow \frac{2}{\pi \cdot XX \cdot YY}$$

$$B \leftarrow \ln\left[\sqrt{\frac{(1 + XX^2) \cdot (1 + YY^2)}{1 + XX^2 + YY^2}}\right]$$

$$C \leftarrow XX \cdot (1 + YY^2)^{\frac{1}{2}} \cdot atan\left[\frac{XX}{(1 + YY^2)^{\frac{1}{2}}}\right]$$

$$D \leftarrow YY \cdot (1 + XX^2)^{\frac{1}{2}} \cdot atan\left[\frac{YY}{(1 + XX^2)^{\frac{1}{2}}}\right]$$

$$E \leftarrow XX \cdot atan(XX)$$

$$F \leftarrow YY \cdot atan(YY)$$

$$F_{12} \leftarrow A \cdot (B + C + D - E - F)$$

Ex: $F12\_parallel\_rectangles(0.5, 1, 1) = 0.117$

===

2. **View factors for coaxial, parallel disks: (See the fig. above for nomenclature)**

Define:

$$R_i = \frac{r_i}{L} \qquad\qquad R_j = \frac{r_j}{L} \qquad\qquad S(R_i, R_j) := 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij}(R_i, R_j) := \frac{1}{2} \cdot \left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4 \cdot \left(\frac{R_j}{R_i}\right)^2 \right]^{\frac{1}{2}} \right] \qquad \text{...View factor for coaxial parallel disks}$$

Now, to plot the graphs:

$R_i := 10, 9.9 .. 0.1$ ...define a range variable

View factor for coaxial parallel disks

Legend:
- $r_j/L = 0.3$
- $r_j/L = 0.6$
- $r_j/L = 1.0$
- $r_j/L = 1.5$
- $r_j/L = 2.0$
- $r_j/L = 4.0$
- $r_j/L = 8.0$

===
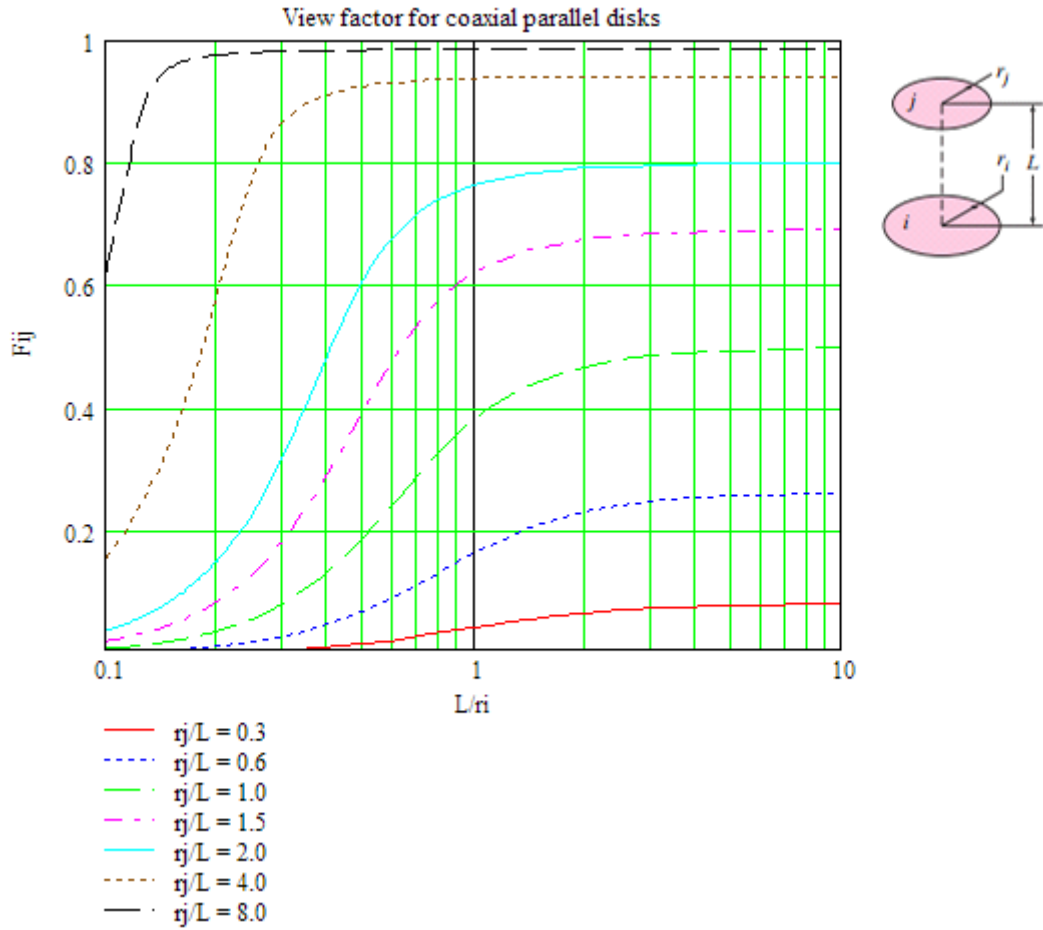
We write the Mathcad Function for F12 for coaxial, parallel disks as follows:

Here,

Inputs are: r_i, r_j and L.

Output is: F12.

$$F12\_parallel\_disks\left(r_i, r_j, L\right) := \begin{vmatrix} R_i \leftarrow \dfrac{r_i}{L} \\[2ex] R_j \leftarrow \dfrac{r_j}{L} \\[2ex] S \leftarrow 1 + \dfrac{1 + R_j^2}{R_i^2} \\[2ex] F12 \leftarrow \dfrac{1}{2}\cdot\left[S - \left[S^2 - 4\cdot\left(\dfrac{r_j}{r_i}\right)^2 \right]^{\frac{1}{2}} \right] \end{vmatrix}$$

Ex: $F12\_parallel\_disks(0.5, 0.6, 1) = 0.232$
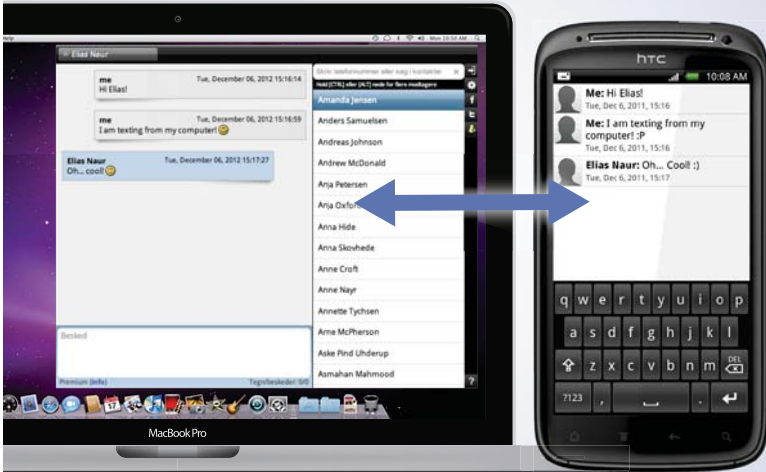
==

3. **View factors for perpendicular rectangles with a common edge: (See the fig. above for nomenclature)**

Define:

$$H = \frac{Z}{X} \quad W = \frac{Y}{X} \quad A(W) := \frac{1}{\pi \cdot W} \quad B(W) := W \cdot \operatorname{atan}\left(\frac{1}{W}\right)$$

$$C(H) := H \cdot \operatorname{atan}\left(\frac{1}{H}\right) \quad D(H, W) := \left(H^2 + W^2\right)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{1}{\left(H^2 + W^2\right)^{\frac{1}{2}}}\right]$$

$$E(H, W) := \frac{\left(1 + W^2\right) \cdot \left(1 + H^2\right)}{\left(1 + W^2 + H^2\right)} \cdot \left[\frac{W^2 \cdot \left(1 + W^2 + H^2\right)}{\left(1 + W^2\right) \cdot \left(W^2 + H^2\right)}\right]^{W^2} \cdot \left[\frac{H^2 \cdot \left(1 + H^2 + W^2\right)}{\left(1 + H^2\right) \cdot \left(H^2 + W^2\right)}\right]^{H^2}$$

$$F_{ij}(H, W) := A(W) \cdot \left(B(W) + C(H) - D(H, W) + \frac{1}{4} \cdot \ln(E(H, W))\right)$$

...View factor for coaxial perpendicular rectangles with a comon edge

Now, to plot the graphs:

$H := 0.1, 0.2.. 10$ ....define a range variable

View factor for perpendicular rectangles

Legend:
— $Y/X = 0.02$
···· $Y/X = 0.05$
—· $Y/X = 0.1$
—·· $Y/X = 0.2$
— $Y/X = 0.6$
···· $Y/X = 1.0$
—· $Y/X = 2.0$
—·· $Y/X = 4.0$
— $Y/X = 10.0$

===
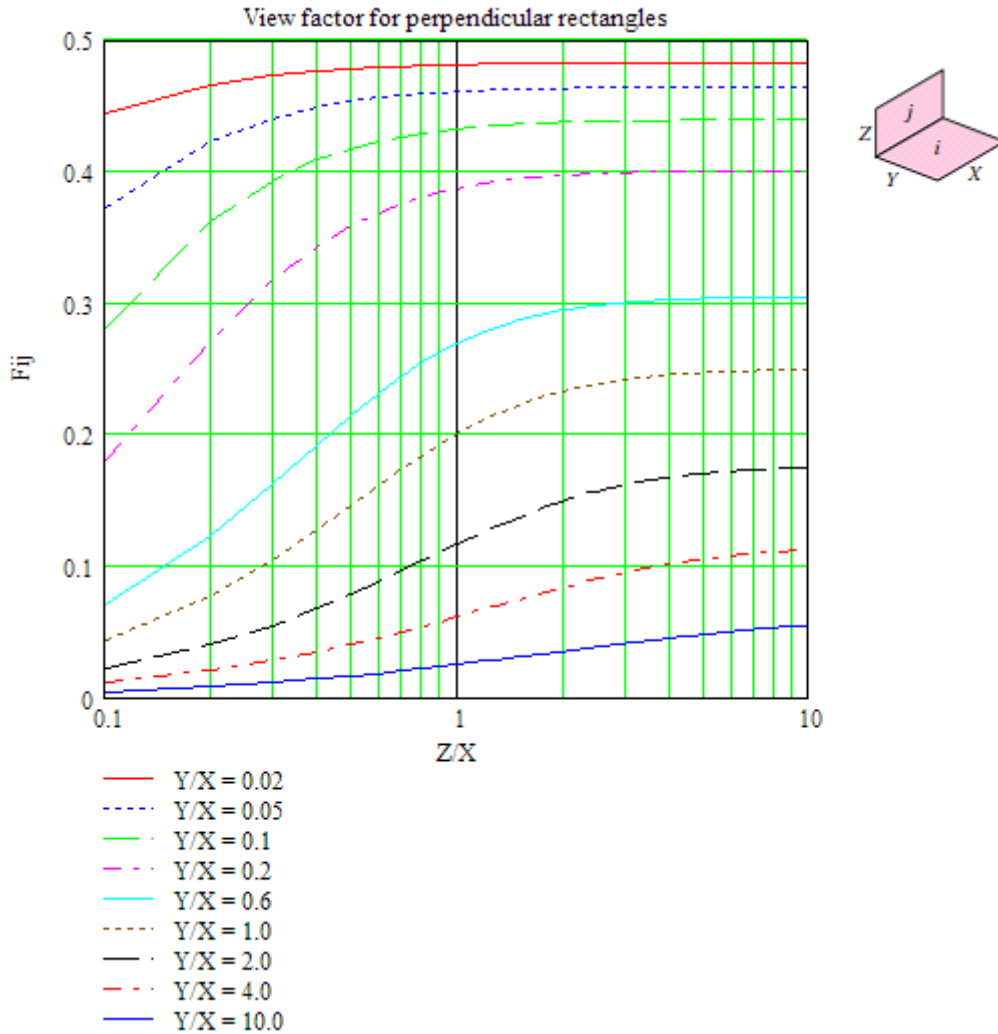
We write the Mathcad Function for F12 for perpendicular rectangles with a common edge as follows:

Here,

Inputs are: X, Y and Z.

Output is: F12.

$$
\text{F12\_perpendicular\_rectangles}\,(X,Y,Z) :=
\begin{aligned}
&H \leftarrow \frac{Z}{X} \\[4pt]
&W \leftarrow \frac{Y}{X} \\[4pt]
&A \leftarrow \frac{1}{\pi \cdot W} \\[4pt]
&B \leftarrow W \cdot \operatorname{atan}\left(\frac{1}{W}\right) \\[4pt]
&C \leftarrow H \cdot \operatorname{atan}\left(\frac{1}{H}\right) \\[4pt]
&D \leftarrow \left(H^2 + W^2\right)^{\frac{1}{2}} \cdot \operatorname{atan}\left[\frac{1}{\left(H^2 + W^2\right)^{\frac{1}{2}}}\right] \\[8pt]
&E \leftarrow \frac{\left(1 + W^2\right)\cdot\left(1 + H^2\right)}{\left(1 + W^2 + H^2\right)} \cdot \left[\frac{W^2 \cdot\left(1 + W^2 + H^2\right)}{\left(1 + W^2\right)\cdot\left(W^2 + H^2\right)}\right]^{W^2} \cdot \left[\frac{H^2 \cdot\left(1 + H^2 + W^2\right)}{\left(1 + H^2\right)\cdot\left(H^2 + W^2\right)}\right]^{H^2} \\[8pt]
&F12 \leftarrow A \cdot \left(B + C - D + \frac{1}{4}\cdot\ln(E)\right)
\end{aligned}
$$

Ex: F12_perpendicular_rectangles(5,5,5) = 0.2

===

Prob.5B.2. Write Mathcad Functions for view factors of following 2_D geometries:

1) Parallel plates with mid-lines connected by perpendicular line
2) Inclined plates of equal width and with a common edge
3) Perpendicular plates with a common edge
4) Three sided enclosure
5) Infinite plane and row of cylinders

Geometries and view factor relations, given at the beginning of the chapter, are reproduced below: (Ref: Cengel)

View factors for few two-dimensional geometries, infinitely long:

Geometry	Relation
Parallel plates with midlines connected by perpendicular line	$W_i = w_i/L$ and $W_j = w_j/L$ $F_{i \to j} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined plates of equal width and with a common edge	$F_{i \to j} = 1 - \sin \dfrac{1}{2}\alpha$
Perpendicular plates with a common edge	$F_{i \to j} = \dfrac{1}{2}\left\{1 + \dfrac{w_j}{w_i} - \left[1 + \left(\dfrac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$
Three-sided enclosure	$F_{i \to j} = \dfrac{w_i + w_j - w_k}{2w_i}$
Infinite plane and row of cylinders	$F_{i \to j} = 1 - \left[1 - \left(\dfrac{D}{s}\right)^2\right]^{1/2}$ $+ \dfrac{D}{s} \tan^{-1}\left(\dfrac{s^2 - D^2}{D^2}\right)^{1/2}$

Mathcad Functions:

(a) For parallel plates with mid-lines connected by perpendicular:

$$F_{ij\_parallel\_plates}(w_i, w_j, L) := \left| \begin{array}{l} W_i \leftarrow \dfrac{w_i}{L} \\[2mm] W_j \leftarrow \dfrac{w_j}{L} \\[4mm] F_{ij} \leftarrow \dfrac{\left[(W_i + W_j)^2 + 4\right]^{\frac{1}{2}} - \left[(W_j - W_i)^2 + 4\right]^{\frac{1}{2}}}{2 \cdot W_i} \end{array} \right.$$

Example:

$$F_{ij\_parallel\_plates}(0.25, 0.5, 0.15) = 0.835$$

(b) For inclined parallel plates of equal width and with a common edge:

$$F_{ij\_inclined\_plates}(\alpha) := 1 - \sin\left(\frac{\alpha \cdot \pi}{180}\right)$$ **Note: α in degrees**

Example: If the angle between plates is 30 deg.:

$$F_{ij\_inclined\_plates}(30) = 0.5$$

(c) For perpendicular plates with a common edge:

$$F_{ij\_perpendicular\_plates}(w_i, w_j) := \frac{1}{2}\cdot\left[1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{\frac{1}{2}}\right]$$

Example: when $w_i = 0.5$ m, $w_j = 0.5$ m:

$$F_{ij\_perpendicular\_plates}(0.5, 0.5) = 0.293$$

(d) For three sided enclosure:

$$F_{ij\_three\_sided\_enclosure}(w_i, w_j, w_k) := \frac{w_i + w_j - w_k}{2\cdot w_i}$$

Example:

$$F_{ij\_three\_sided\_enclosure}(0.7, 0.7, 0.5) = 0.643$$

(e) For infinite plane and row of cylinders:

$$F_{ij\_infinite\_plane\_to\_row\_of\_cyl}(D, s) := 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{\frac{1}{2}} + \frac{D}{s}\cdot atan\left(\frac{s^2 - D^2}{D^2}\right)^{\frac{1}{2}}$$

Example: If D = 1 cm, s = 5 cm:

$$F_{ij\_infinite\_plane\_to\_row\_of\_cyl}(0.01, 0.05) = 0.268$$

===

Prob.5B.2. Find out the net heat transferred between two circular disks 1 and 2, oriented one above the other, parallel to each other on the same centre line, as shown in Fig. Disk 1 has a radius of 0.5 m and is maintained at 1000 K, and disk 2 has a radius of 0.6 m and is maintained at 600 K. Assume both the disks to be black surfaces.
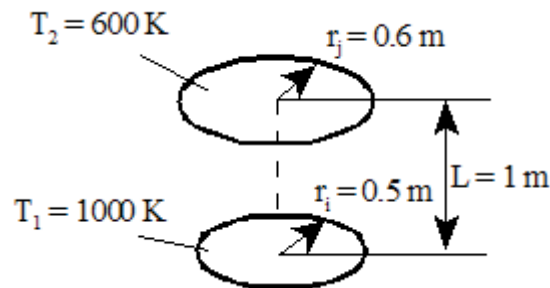
Fig. Prob.5B.2

Mathcad Solution:

Data:

$r_i := 0.5$ m....radius of disk 1

$r_j := 0.6$ m....radius of disk 2

$L := 1$ m....distance between disk 1 and disk 2

$T_1 := 1000$ K....temp. of disk 1

$T_2 := 600$ K...temp. of disk 2

$\sigma := 5.67 \cdot 10^{-8}$ W/m².K....Stefan-Boltzmann const.

$A_1 := \pi \cdot r_i^2$ i.e. $A_1 = 0.785$ m²....area of disk 1

$A_2 := \pi \cdot r_j^2$ i.e. $A_2 = 1.131$ m²....area of disk 2

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40):

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{W....(13.40)}$$

So, the problem reduces to calculating the view factor F_{12} or F_{21} .

We can easily find out F12 using the graphs given earlier. However, we can determine F12 analytically *more accurately* with Mathcad Functions written above.

We use the Mathcad Function already written above:

$$F12\_parallel\_disks\left(r_i, r_j, L\right) = 0.232 \quad \text{...View factor for coaxial parallel disks}$$

(Note: verify this result by referring to the graph.)

i.e. $\quad F_{12} := 0.232$

Therefore, net heat transfer between disks 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{W...from eqn. (13.40)}$$

i.e. $\quad Q_{net} = 8.992 \times 10^3 \quad$ **W....Ans.**

==

Prob.5B.3. Find out the net heat transferred between two aligned parallel rectangles, as shown in Fig. Surface 1 is maintained at 500 K, and surface 2 is maintained at 1000 K. Assume both the surfaces to be black surfaces.
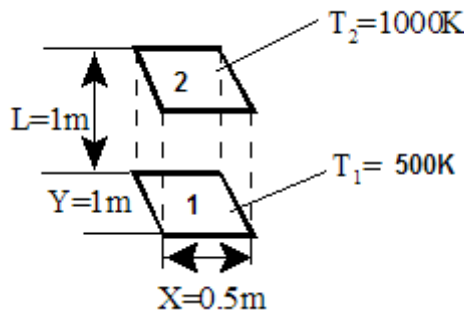
Fig.Prob.5B.3

Mathcad Solution:

Data:

$X := 0.5$ m....horizontal x-dimension of surfaces 1 and 2

$Y := 1.0$ m....horizontal y-dimension of surfaces 1 and 2

$L := 1.0$ ...vertical distane between surfaces 1 and 2

$T_1 := 500$ K....temp. of surface 1

$T_2 := 1000$ K...temp. of surface 2

$\sigma := 5.67 \cdot 10^{-8}$ W/m^2.K....Stefan-Boltzmann const.

Calculations:

$A_1 := X \cdot Y$ i.e. $A_1 = 0.5$ m^2....area of surface 1

$A_2 := X \cdot Y$ i.e. $A_2 = 0.5$ m^2....area of surface 2

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40), viz.

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{W....(13.40)}$$

So, the problem reduces to calculating the view factor F_{12} or F_{21}. We can find out F_{12} using the graphs. However, we can determine F_{12} analytically *more accurately* with Mathcad Function using the view factor relation given in Table for aligned parallel rectangles.

We use the the Mathcad Function already written above for view factor as follows:

$$F12\_parallel\_rectangles(X, Y, L) = 0.117$$

(Note: verify this result from the graph.)

i.e. $F_{12} := 0.117$

Therefore, net heat transfer between surfaces 1 and 2:

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{W...from eqn. (13.40)}$$

i.e. $Q_{net} = -3.11 \times 10^3 \quad$ **W....Ans.**

Note: -ve sign indicates that net heat transfer is to the surface 1, since surface 2 is at a higher temperature than surface 1.

==

Prob.5B.4. Find out the net heat transferred between the areas A1 and A2 shown in Fig. Area 1 is maintained at 900 K, and area 2 is maintained at 400 K. Assume both the surfaces to be black.
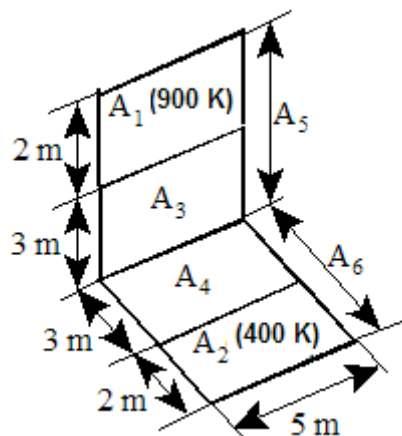
Fig.Prob.5B.4

Mathcad Solution:

This is the case of heat transfer between two black surfaces. So, we use eqn. (13.40), viz.

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) = A_2 \cdot F_{21} \cdot \sigma \cdot \left(T_1^4 - T_2^4\right) \quad \text{W....(13.40)}$$

So, the problem reduces to calculating the view factor F_{12} or F_{21}.

We see that to calculate F_{12} for areas A_1 and A_2 as oriented in the Fig., we do not readily have an analytical relation or a graph.

Let us denote the combined areas $(A_1 + A_3)$ by A_5 and $(A_2 + A_4)$ by A_6.

Then, we see that A_5 and A_6 are *perpendicular rectangles which have a common edge*, and we have graphs or analytical relation for the view factor for such an orientation.

Then, we resort to **view factor algebra**, as follows:

Remember that by definition, **view factor F_{12} is the fraction of radiant energy emitted by surface 1 which falls directly on surface 2.** Looking at the fig., we can say that fraction of energy leaving A_1 and falling on A_2 is equal to the fraction falling on A_6 *minus* the fraction falling on A_4.

i.e. $\quad F_{12} = F_{16} - F_{14} \qquad$ ...by definition of view factor

i.e. $\quad F_{12} = F_{61} \cdot \dfrac{A_6}{A_1} - F_{41} \cdot \dfrac{A_4}{A_1} \qquad$ ...since by reciprocity relation, $A_1 . F_{16} = A_6 . F_{61}$, and $A_1 . F_{14} = A_4 . F_{41}$.

i.e. $\quad F_{12} = \dfrac{A_6}{A_1} \cdot \left(F_{65} - F_{63} \right) - \dfrac{A_4}{A_1} \cdot \left(F_{45} - F_{43} \right) \qquad$ eqn.(A)....using the definition of view factor, as done in first step above

Now, observe that view factors F_{65}, F_{63}, F_{45} and F_{43} refer to perpendicular rectangles
with a comon edge, and can be readily obtained from the graphs, or by analytical relation
given in Table.

**We have already written Mathcad Function for view factor relation for perpendicular rectangles
with a common edge (See Prob.5B.1).**

To find F_{65}:

$X := 5 \quad Y := 5 \quad Z := 5 \quad$ ....See. Fig.

$\text{F12\_perpendicular\_rectangles}(X, Y, Z) = 0.2 \quad$ **....From the Mathcad Function**

i.e. $\quad F_{65} := 0.2 \quad$ ...view factor from area A_6 to A_5

Note: This value can be verified from graphs also.

To find F_{63}:

$X := 5 \quad Y := 5 \quad Z := 3 \quad$ ...See. Fig.

$\text{F12\_perpendicular\_rectangles}(X, Y, Z) = 0.161 \quad$ **....From the Mathcad Function**

i.e. $\quad F_{63} := 0.161 \quad$ ...view factor from area A_6 to A_3

Note: This value also can be verified from graphs.

To find F_{45}:

$X := 5 \quad Y := 3 \quad Z := 5 \quad$ ..See Fig.

$\text{F12\_perpendicular\_rectangles}(X, Y, Z) = 0.269 \quad$ **....From the Mathcad Function**

i.e. $\quad F_{45} := 0.269 \quad$ ...view factor from area A_4 to A_5

Note: This value also can be verified fromgraphs.

To find F_{43}:

$X := 5 \quad Y := 3 \quad Z := 3 \quad$ ..See Fig.

$\text{F12\_perpendicular\_rectangles}(X, Y, Z) = 0.231 \quad$ **....From the Mathcad Function**

i.e. $\quad F_{43} := 0.231 \quad$ ...view factor from area A_4 to A_3

Note: This value also can be verified fromgraphs.

Areas:

From Fig. we have:

$A_1 := 10 \qquad A_2 := 10 \qquad A_3 := 15 \qquad A_4 := 15 \quad m^2$

$A_5 := 25 \qquad A_6 := 25 \quad m^2$

Then, from eqn.(A):

$$F_{12} := \left[\frac{A_6}{A_1} \cdot (F_{65} - F_{63}) - \frac{A_4}{A_1} \cdot (F_{45} - F_{43}) \right]$$

i.e. $F_{12} = 0.041$ ....view factor from A_1 to A_2.....Ans.

Note: F_{21} can be calculated, if required, by reciprocity relation, i.e. $A_1 \cdot F_{12} = A_2 \cdot F_{21}$

Therefore, net heat transfer between surfaces 1 and 2:

$$Q_{net} = A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4 \right) \quad W...from\ eqn.\ (13.40)$$

Here, we have:

$T_1 := 900 \qquad K....temp.\ of\ surface\ A_1$

$T_2 := 400 \qquad K...temp.\ of\ surface\ A_2$

$\sigma := 5.67 \cdot 10^{-8} \quad W/m^2.K....Stefan\text{-}Boltzmann\ const.$

Therefore,

$$Q_{net} := A_1 \cdot F_{12} \cdot \sigma \cdot \left(T_1^4 - T_2^4 \right)$$

i.e. $Q_{net} = 1.448 \times 10^4 \qquad W....Ans.$

===

Prob.5B.5. Find out the relevant view factors for the geometries shown in Fig. below:

    a) a long tube with cross-section of an equilateral triangle

    b) a blackbody completely enclosed by another black body

    c) diagonal partition inside a long square duct

    d) sphere of diameter d inside a cubical box of sides, L = d

    e) hemispherical surface closed by a plane surface, and

    f) the end and surface of a circular cylinder whose length is equal to diameter.
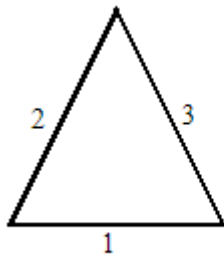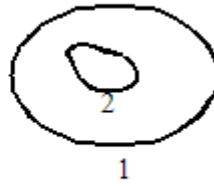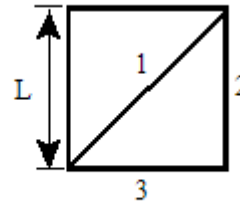
(a) Equilateral triangle (b) A blackbody completely enclosed by another black body (c) Diagonal partition in a long square duct
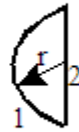
(d) Sphere inside a cubical box (e) Hemispherical bowl (f) Cylinder with length = diameter
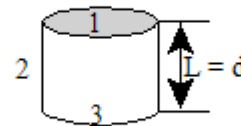
Fig.Prob.5B.5

General principle in solving these problems is to invoke: Summation rule, reciprocity theorem, inspection of geometry for symmetry, and of course, remembering the definition of view factor:

(a) Long tube wih cross-section of equilateral triangle: See Fig. (a)

For surface 1: $F_{11} + F_{12} + F_{13} = 1$ ....Summation rule

But, $F_{11} = 0$ ...since surface 1 is flat and can not see itself.

Therefore, $F_{12} + F_{13} = 1$

Now, *by inspection of geometry*, we find that surfaces 2 and 3 are located *symmetrically w.r.t. surface 1*, since it is an equilateral triangle. Therefore, radiation from surface 1 is divided equally between surfaces 2 and 3.

i.e. $F_{12} = F_{13} = 0.5$

Similarly, for surface 2, we write:

$$F_{21} + F_{23} = 1$$

i.e. $F_{23} = 1 - F_{21}$

But, $F_{21} = \dfrac{A_1}{A_2} \cdot F_{12} = F_{12}$ ...since $A_1 = A_2$

Therefore, $F_{23} = 1 - F_{12} = 0.5$

Similarly for surface 3.

(b) Black body enclosed inside a black enlosure: See Fig. (b)

For surface 1: $F_{11} + F_{12} = 1$ ...by Summation rule

and, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ ...by reciprocity

i.e. $F_{12} = \dfrac{A_2}{A_1} \cdot F_{21}$

Now, $F_{11} = 1 - F_{12}$

i.e. $F_{11} = 1 - \dfrac{A_2}{A_1} \cdot F_{21}$

But, $F_{21} = 1$ since all the energy radiated by surface 2 is directly intercepted by surface 1.

Therefore, $F_{11} = 1 - \dfrac{A_2}{A_1}$

(c) Diagonal partition within a long square duct: See Fig. (c)

For surface 1: $F_{11} + F_{12} + F_{13} = 1$ ...by Summation rule

But, $F_{11} = 0$ ..since surface 1 is flat and can not 'see' itself.

Therefore, $F_{12} + F_{13} = 1$

By symmetry: $F_3 = F_{12} = 0.5$ ..since radiation emitted by surface 1 is divided equally between surfaces 2 and 3

By reciprocity: $F_{21} = \dfrac{A_1}{A_2} \cdot F_{12}$

i.e. $F_{21} = \dfrac{\sqrt{2} \cdot L}{L} \cdot 0.5 = 0.71$

(d) Sphere inside a cubical box: See Fig. (d)

For surface 1: $F_{11} = 0$ ...since surface of the sphere is convex and can not 'see' itself.

and, $F_{11} + F_{12} = 1$ ...by Summation rule

Therefore, $F_{12} = 1$

By reciprocity: $F_{21} = \dfrac{A_1}{A_2} \cdot F_{12}$

i.e. $F_{21} = \dfrac{\pi \cdot d^2}{6 \cdot d^2} = \dfrac{\pi}{6}$ ...since L = d

i.e. $F_{21} = 0.524$

(e) Hemispherical surface closed by a flat surface: See Fig. (e)

For surface 1: $\quad F_{11} + F_{12} = 1 \quad$ ....Summation rule

Also, $\quad F_{21} = 1 \quad$ ...since surface 2 is flat and can not 'see' itself, and all radiation emitted by surface 2 falls directly on the hemispherical surface 1.

By reciprocity: $\quad F_{12} = \dfrac{A_2}{A_1} \cdot F_{21}$

i.e. $\quad F_{12} = \dfrac{\pi \cdot r^2}{2 \cdot \pi \cdot r^2} \cdot 1 = 0.5$

Therefore, $\quad F_{11} = 1 - F_{12} = 0.5$

(f) End and sides of a circular cylinder (L = d): See Fig. (f)

From the fig., note that the two end surfaces are denoted by 1 and 3 and the side surface is denoted by 2.

<u>View factor F_{13}:</u> Surfaces 1 and 3 can be considered as two concentric parallel disks.
Therefore, F_{13} can be found out from graphs or by analytical relation given in Table. Let us
use the analytical relation:

We have:

$$R_i = \frac{r_i}{L} \qquad\qquad R_j = \frac{r_j}{L} \qquad\qquad S(R_i, R_j) := 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij}(R_i, R_j) := \frac{1}{2} \cdot \left[S(R_i, R_j) - \left[S(R_i, R_j)^2 - 4 \cdot \left(\frac{R_j}{R_i}\right)^2 \right]^{\frac{1}{2}} \right]$$

...View factor for coaxial parallel disks

Now, for a cylinder with L = d:

$$R_i := 0.5 \qquad R_j := 0.5$$

Therefore, $\quad F_{ij}(R_i, R_j) = 0.172$

i.e. $\quad F_{13} := 0.172 \quad$ ...view factor from surface 1 to surface 3

For surface 1: $\quad F_{11} + F_{12} + F_{13} = 1 \quad$ ...by Summation rule

But, $\quad F_{11} = 0 \quad$ ...since surface 1 is flat and can not 'see' itself.

Therefore, $\quad F_{12} = 1 - F_{13}$

i.e. $\quad F_{12} = 0.828$

By reciprocity: $\quad F_{21} = \frac{A_1}{A_2} \cdot F_{12}$

i.e. $\quad F_{21} = \frac{\frac{\pi \cdot d^2}{4}}{\pi \cdot d \cdot L} \cdot 0.828$

i.e. $\quad F_{21} = \frac{\frac{\pi \cdot d^2}{4}}{\pi \cdot d^2} \cdot 0.828 \quad$ ...since L = d

i.e. $\quad F_{21} = \frac{0.828}{4} = 0.207$

Also, by symmetry: $F_{32} = F_{12} = 0.828$

and, $F_{23} = F_{21} = 0.207$

===

Prob.5B.6. Find out the view factor (F_{11}) of a cavity with respect to itself. Hence, find out the view factor F_{11} for the following:

    a) a cylindrical cavity of diameter 'd' and depth 'h'
    b) a conical cavity of diameter 'd' and depth 'h'
    c) a hemispherical bowl of diameter 'd'
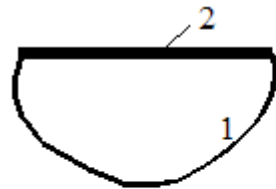
(a) Cavity (1) closed by a   (b) Cylindrical cavity
hypothetical flat surface (2)
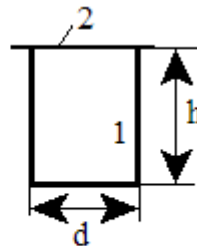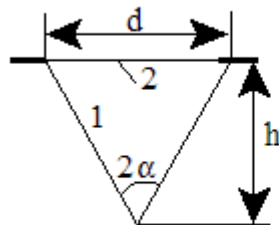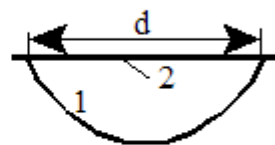
(c) Conical cavity   (d) Hemispherical bowl

Fig.Prob.5B.6

Mathcad Solution:

View factor of a general cavity w.r.t. itself: See Fig.(a) above.

We desire to find F_{11}. It is obvious from the fig. that part of the radiation emitted by the cavity surface 1, falls on itself and therefore, F_{11} exists.

Close the opening (or mouth) of the cavity by a hypothetical flat surface 2. Then, surfaces 1 and 2 together form an enclosure. We can write:

For surface 1: $F_{11} + F_{12} = 1$ ...(a)....by Summation rule

For surface 2: $F_{21} + F_{22} = 1$ ....by Summation rule

But, $F_{22} = 0$ ..since surface 2 is flat and can not 'see' itself.

Therefore, $F_{21} = 1$

Further, $A_1 \cdot F_{12} = A_2 \cdot F_{21}$ ....by reciprocity

i.e. $F_{12} = \dfrac{A_2}{A_1}$

Now, $F_{11} = 1 - F_{12}$ ...from eqn.(a)

i.e. $F_{11} = 1 - \dfrac{A_2}{A_1}$ .....eqn.(b)
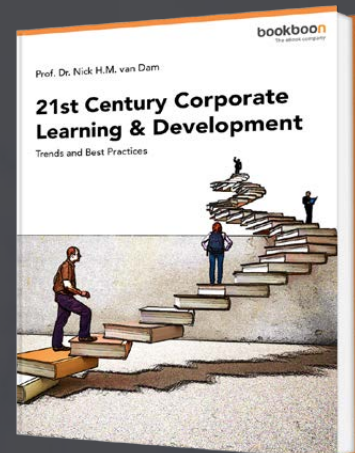
Eqn.(b) is an important result, since it gives the shape factor of any general cavity w.r.t. itself.

Now, this result will be applied to following specific cavities:

(a) F_{11} for a cylindrical cavity of diameter 'd' and depth 'h': See Fig.(b)

We have: $F_{11} = 1 - \dfrac{A_2}{A_1}$

i.e. $F_{11} = 1 - \dfrac{\dfrac{\pi \cdot d^2}{4}}{\dfrac{\pi \cdot d^2}{4} + \pi \cdot d \cdot h}$ ...Note that A_1 consists of the area of bottom circular surface and the cylindrical side surfaces

i.e. $F_{11} = \dfrac{4 \cdot h}{4 \cdot h + d}$ ....Ans.

(b) F_{11} for a conical cavity of diameter 'd' and depth 'h': See Fig. (c)

We have: $F_{11} = 1 - \dfrac{A_2}{A_1}$

i.e. $F_{11} = 1 - \dfrac{\dfrac{\pi \cdot d^2}{4}}{\dfrac{\pi \cdot d \cdot L}{2}}$ ...where L is the slant height of the cone

i.e. $F_{11} = 1 - \dfrac{d}{2 \cdot L}$

i.e. $F_{11} = 1 - \sin(\alpha)$ ...where α is the half-vertex angle of the cone.

Alternatively: To get F_{11} in terms of depth 'h', we can write:

We have: $F_{11} = 1 - \dfrac{d}{2 \cdot L}$

i.e. $F_{11} = 1 - \dfrac{d}{2 \cdot \sqrt{h^2 + \dfrac{d^2}{4}}}$

i.e. $F_{11} = 1 - \dfrac{d}{\sqrt{4 \cdot h^2 + d^2}}$

(c) F_{11} for a hemispherical bowl of diameter 'd' : See Fig.(d)

We have: $F_{11} = 1 - \dfrac{A_2}{A_1}$

i.e. $F_{11} = 1 - \dfrac{\dfrac{\pi \cdot d^2}{4}}{\dfrac{\pi \cdot d^2}{2}}$

i.e. $F_{11} = 1 - \dfrac{1}{2}$

i.e. $F_{11} = 0.5$ ...Ans.

This result means that for any hemispherical cavity, half of the radiation emitted by the surface 1 falls on itself; it also means that the remaining half falls on the closing surface 2.

===

Prob.5B.7. A 70 mm thick metal plate has a hole of 30 mm dia drilled through it. If the plate is held at 300 C and the surrounding is at 27 C, find the energy escaping to the surroundings from both ends of the hole. Assume the metal and surroundings as black bodies.

Mathcad Solution:
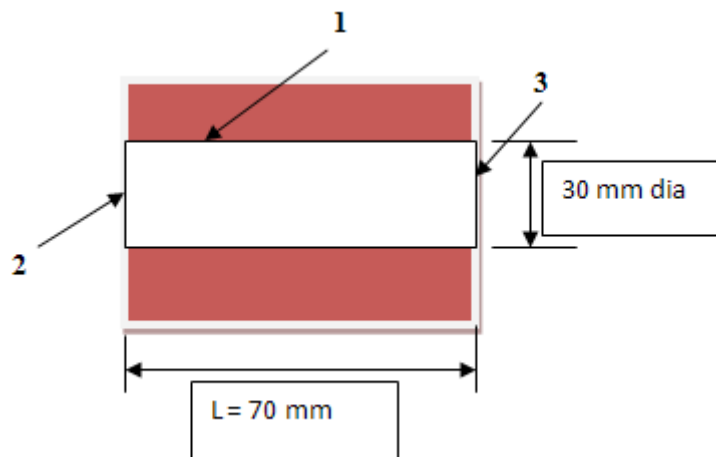
Fig.Prob.5B.7

Data:

$$r_i := 0.015 \quad m \qquad r_j := 0.015 \quad m \qquad L := 0.07 \quad m$$

$$A1 := \pi \cdot 2 \cdot r_i \cdot L \qquad i.e. \qquad A1 = 6.597 \times 10^{-3} \quad m^2$$

$$A2 := \pi \cdot r_i^2 \qquad i.e. \qquad A2 = 7.069 \times 10^{-4} \quad m^2$$

$$A3 := \pi \cdot r_j^2 \qquad i.e. \qquad A3 = 7.069 \times 10^{-4} \quad m^2$$

$$T1 := 300 + 273 \quad K \qquad T_a := 27 + 273 \quad K$$

$$\sigma := 5.67 \cdot 10^{-8} \quad W/m^2.K^4......\text{Stefan Boltzmann constant}$$

Calculations:

$$F_{22} + F_{23} + F_{21} = 1 \quad \text{..eqn (a)....by summation rule}$$

$$F_{22} := 0 \quad \text{...for a flat surface 2}$$

F23 is the view factor from surface 2 to surface 3, i.e. between two coaxial disks

Let us apply the Mathcad Function already written to calculate F23.

$$F_{23} := F12\_parallel\_disks(r_i, r_j, L) \quad \text{...using Mathcad Function}$$

i.e. $F_{23} = 0.042$

Then, from eqn. (a): $\quad F_{21} := 1 - (F_{22} + F_{23})$

i.e. $\quad F_{21} = 0.958$

Now, applying reciprocal rule between surfaces 1 and 2:

$$A1 \cdot F_{12} = A2 \cdot F_{21} \quad \text{...by reciprocity}$$

Therefore: $\quad F_{12} := \dfrac{A2 \cdot F_{21}}{A1}$

i.e. $\quad F_{12} = 0.103$

Now, by symmetry: $\quad F_{13} := F_{12}$

Therefore: Heat lost by both ends:

$$Q_{tot} := 2 \cdot A1 \cdot F_{12} \cdot \sigma \cdot \left(T1^4 - T_a^4 \right) \qquad W$$

i.e. $\quad \boxed{Q_{tot} = 7.655} \quad \boxed{\text{W....Ans.}}$

===

EES Functions to determine View Factors:

Let us write EES Functions to find out View Factors for **three practically important geometries**:

    a) Aligned, parallel rectangles,

    b) Coaxial, parallel disks, and

    c) Perpendicular rectangles with a common edge

----

    a) **Aligned, parallel rectangles:**
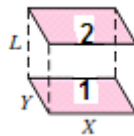
$UnitSystem SI Pa J Radians

FUNCTION F12_parallel_rectangles(X, Y, L)

{Gives View factor for coaxial, parallel rectangles separated by a distance L
Note: Trigonometric functions are used with Radians; so, set the angles to Radians in Unit Systems}

XX := X / L

YY := Y / L

A := 2 / (pi * XX * YY)

B := ln(sqrt((1 + XX^2) * (1 + YY^2) / (1 + XX^2 + YY^2)))

C := XX * (1 + YY^2)^ (1/2) * arctan(XX / (1 + YY^2)^(1/2))

D := YY * (1 + XX^2)^(1/2) * arctan(YY / (1 + XX^2)^(1/2))

E := XX * arctan(XX)

F := YY * arctan(YY)

F12_parallel_rectangles := A * (B + C + D – E – F)

END

"---"

b) **Coaxial, parallel disks:**
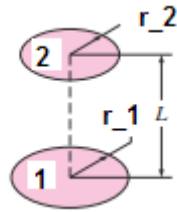
FUNCTION F12_parallel_disks(r_1, r_2, L)

{Gives View factor for coaxial, parallel disks separated by a distance L}

RR_1 := r_1 / L
RR_2 := r_2 / L

S := 1 + (1 + RR_2^2) / RR_1^2

F12_parallel_disks := (1/2) * (S – (S^2 – 4 * (r_2 / r_1)^2)^(1/2))

END

"---"

c) **Perpendicular rectangles with a common edge:**
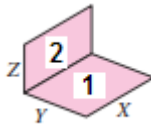
$UnitSystem SI Pa J Radians

FUNCTION F12_perpendicular_rectangles(X, Y, Z)

{Gives View factor for coaxial, perpendicular rectangles with a common edge
Note: Trigonometric functions are used with Radians; so, set the angles to Radians in Unit Systems}

H := Z / X

W := Y / X

A := 1 / (pi * W)

B := W * arctan(1 / W)

C := H * arctan(1 / H)

D := (H^2 + W^2)^(1/2) * arctan(1 / (H^2 + W^2)^(1/2))

E := (1 + W^2) * (1 + H^2) / (1 + W^2 + H^2)

F := (W^2 * (1 + W^2 + H^2) / ((1 + W^2) * (W^2 + H^2)))^(W^2)

G := (H^2 * (1 + H^2 + W^2) / ((1 + H^2) * (H^2 + W^2)))^(H^2)

F12_perpendicular_rectangles := A * (B + C – D + (1/4)*ln(E * F * G))

END

"---"

Now, let us use these EES Functions to solve some problems:

Prob.5B.8. Consider a truncated cone with top and bottom diameters of 10 and 20 cm and a height of 10 cm. Calculate:

      a) the shape factor from the top surface to the sides and also from the side surface to itself.

      b) If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2 (Q_12), if all the surfaces are black.

      c) Plot F_12 and Q_12 for L varying from 0.1 to 1 m
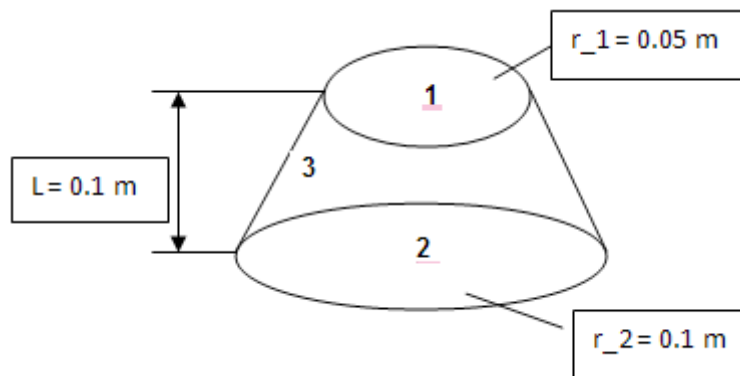
Fig.Prob.5B.8

EES Solution:

Let the top surface be designated as 1, bottom surface as 2, and the side surfaces as 3. See the fig. above.

Then:

"Data:"

r_1 = 0.05 [m]

r_2 = 0.1 [m]

L = 0.1 [m]

"We have to find out F13 and F33:"

"Calculations:"

A1 = pi * r_1^2 "[m^2]…. area of top surface"

A2 = pi * r_2^2 "[m^2]…area of bottom surface"

A3 = pi * (r_1 + r_2) * sqrt((r_2 – r_1)^2 + L^2) "[m^2]…area of the side surface"

"Now:

F11 + F12 + F13 = 0 …… by Summation rule

But, F11 = 0… since it is a flat surface.

We find F12 by the EES Function for parallel disks, written above.

So, we have:

F11 = 0

F12 = F12_parallel_disks(r_1, r_2, L) "using the EES Function for parallel disks"

F11 + F12 + F13 = 1 "….finds the view factor from top surface to side surface, F13"

"Now, by reciprocal relation between surfaces 1 and 3:"

A1 * F13 = A3 * F31 "...finds view factor F31"

"Now, by reciprocal relation between surfaces 1 and 2:"

A1 * F12 = A2 * F21 "...finds view factor from surface 2 to 1, F21"

"And, by reciprocal relation between surfaces 2 and 3:"

F22 = 0 "...since surface 2 is flat"

"For surface 2, by Summation rule:"

F22 + F21 + F23 = 1 "...determines F23"

A2 * F23 = A3 * F32 "...finds view factor from surface 3 to 2, F32"

"Now, applying Summation rule to surface 3:"

F33 + F31 + F32 = 1 "....determines F33"

Results:

Main | F12_parallel_disks

Unit Settings: SI C Pa J mass rad

A1 = 0.007854 [m²]	A2 = 0.03142 [m²]	A3 = 0.05269 [m²]	F11 = 0
F12 = 0.4689	F13 = 0.5311	F21 = 0.1172	F22 = 0
F23 = 0.8828	F31 = 0.07918	F32 = 0.5264	F33 = 0.3944
L = 0.1 [m]	r₁ = 0.05 [m]	r₂ = 0.1 [m]	

Main   F12_parallel_disks

Local variables in Function F12_parallel_disks (1 call, 0.00 sec)

$F12_{parallel,disks} = 0.4689$ $L = 0.1$ [m] $RR_1 = 0.5$

$RR_2 = 1$ $r_1 = 0.05$ [m] $r_2 = 0.1$ [m]

$S = 9$

Thus:

Shape factor from the top surface to the sides = F13 = 0.5311 …. Ans.

Shape factor from the side surface to itself = F33 = 0.3944 …. Ans.

In addition: (b) If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2, if all the surfaces are black.

Add the following code at the end of the above program and run it again:

"If surface 1 is at 1000 K and surface 2 is at 500 K, find the net exchange of heat between 1 and 2, if all the surfaces are black."

T1 = 1000 [K]

T2 = 500 [K]

sigma = 5.67E-08 [W/m^2-K^4] "…Stefan Boltzmann const."

Q_12 = A1 * F12 * sigma * (T1^4 – T2^4) "[W] …. net radiant exchange between surfaces 1 and 2"

We get:

Unit Settings: SI C Pa J mass rad

$A1 = 0.007854$ $[m^2]$

$A2 = 0.03142$ $[m^2]$

$A3 = 0.05269$ $[m^2]$

$F11 = 0$

$F12 = 0.4689$

$F13 = 0.5311$

$F21 = 0.1172$

$F22 = 0$

$F23 = 0.8828$

$F31 = 0.07918$

$F32 = 0.5264$

$F33 = 0.3944$

$L = 0.1$ $[m]$

$Q_{12} = 195.7$ $[W]$

$r_1 = 0.05$ $[m]$

$r_2 = 0.1$ $[m]$

$\sigma = 5.670E\text{-}08$ $[W/m^2 K^4]$

$T1 = 1000$ $[K]$

$T2 = 500$ $[K]$

i.e. $Q\_12 = 195.7$ W … net heat exchange between surfaces 1 and 2 … Ans.

Also, plot F_12 and Q_12 against L, with L varying from 0.1 m to 1 m:

First, compute the Parametric Table. (Here, we have included F_33 also)

Table 1	Table 2			
▶ 1..10	L [m]	F12	F33	Q_{12} [W]
Run 1	0.1	0.4689	0.3944	195.7
Run 2	0.2	0.1922	0.6269	80.26
Run 3	0.3	0.09779	0.7367	40.83
Run 4	0.4	0.05802	0.7981	24.22
Run 5	0.5	0.03811	0.8367	15.91
Run 6	0.6	0.02685	0.8631	11.21
Run 7	0.7	0.0199	0.8822	8.309
Run 8	0.8	0.01533	0.8967	6.399
Run 9	0.9	0.01216	0.908	5.076
Run 10	1	0.009877	0.9171	4.123

Now, plot the results:

View Factor (F_{33}) vs distance between plates (L)

Net heat transfer (Q_{12}) vs distance between plates (L)

===
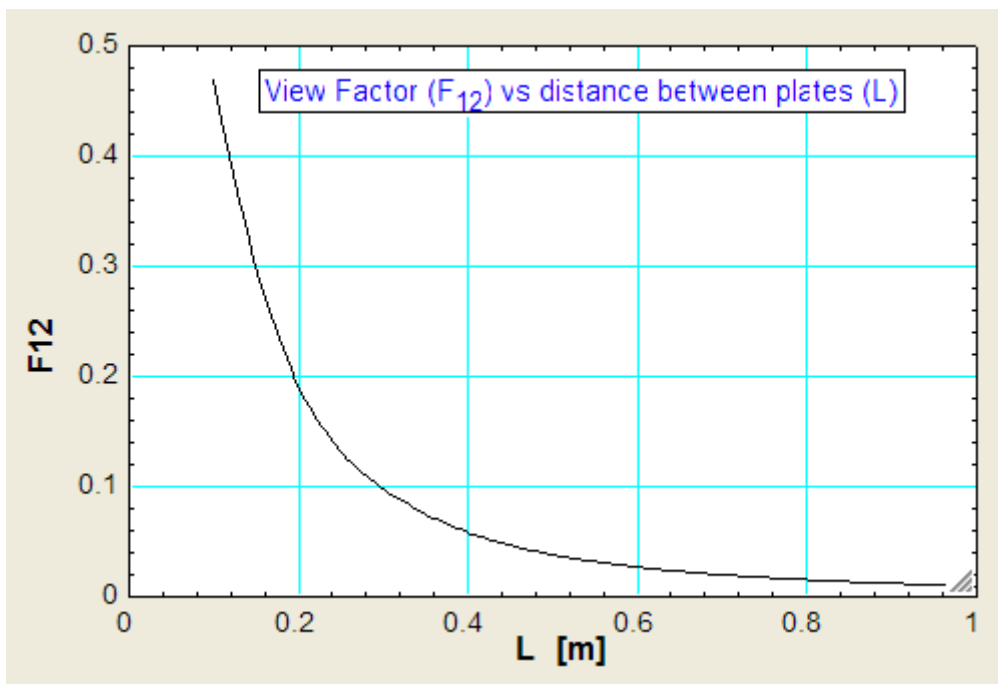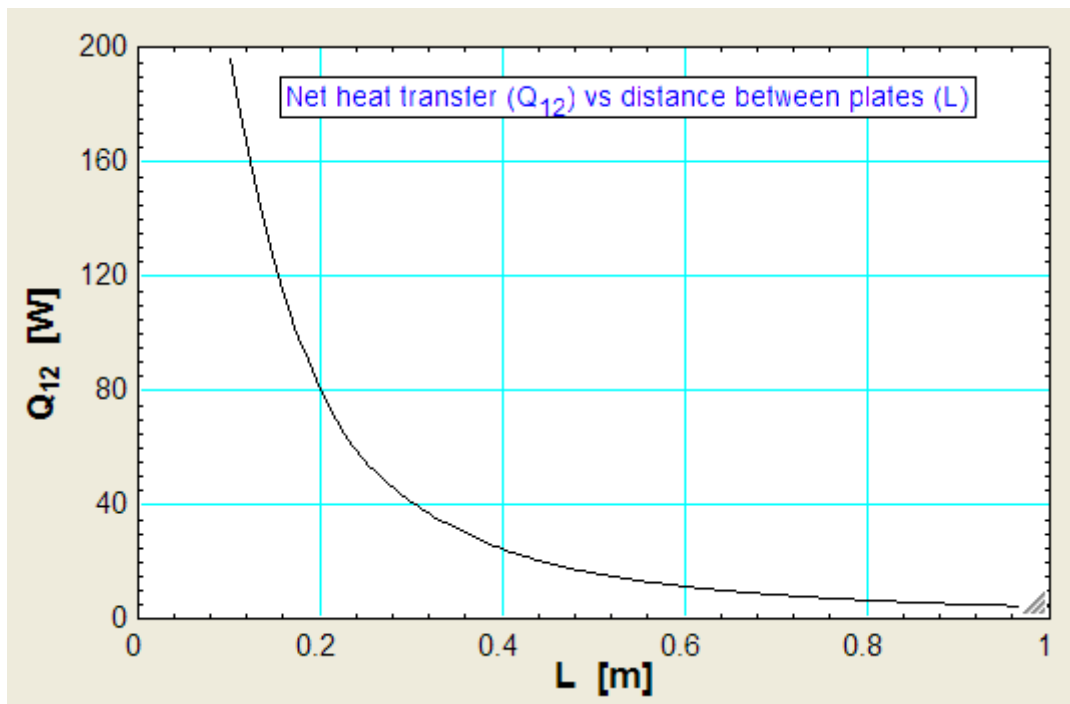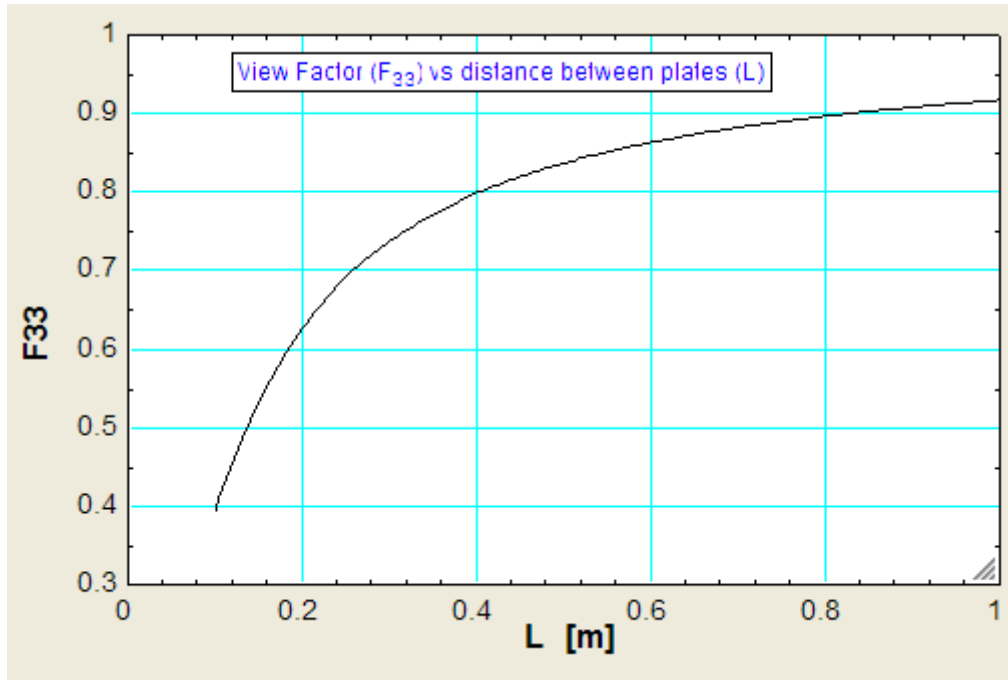
Prob.5B.9. Consider two coaxial, parallel rectangles, each of size: 1 m × 1m, with a distance of 2 m from each other. If surface 1 is at 800 K and the surface 2 at 400 K, find the net heat interchange between the two surfaces. Consider the surfaces as black.
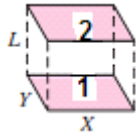
Fig.Prob.5B.9

EES Solution:

"Data:"

X = 1 [m]

Y = 1 [m]

L =2 [m]

T1 = 800 [K]

T2 = 400 [K]

sigma = 5.67E-08[W/m^2-K^4] "…Stefan Boltzmann constant"

"We have to find out F 12:"

"Calculations:"

A1 = X * Y "[m^2]…. area of top surface"

A2 =A1 "[m^2]…. area of bottom surface"

F_12 = F12_parallel_rectangles(X, Y, L)

"Net exchange of heat between A1 and A2:"

Q_12 = A1 * F_12 * sigma * (T1^4 – T2^4) "[W] … net exchange of heat between surfaces1 and 2"

Results:

Unit Settings: SI C Pa J mass rad

A1 = 1 [m^2]	A2 = 1 [m^2]	F_{12} = 0.06859
L = 2 [m]	Q_{12} = 1493 [W]	σ = 5.670E-08 [W/m^{2}K^4]
T1 = 800 [K]	T2 = 400 [K]	X = 1 [m]
Y = 1 [m]		

Thus:

Net exchange of heat between A1 and A2 = Q_12 = 1493 W … Ans.

In addition:

Plot the variation of F_12 and Q_12 as the distance between the plates (L) varies from 0.5 m to 2.5 m:

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First, compute the Parametric Table:

Table 1

	L [m]	F_{12}	Q_{12} [W]
Run 1	0.5	0.4153	9041
Run 2	0.6	0.3545	7718
Run 3	0.7	0.3044	6628
Run 4	0.8	0.263	5726
Run 5	0.9	0.2286	4977
Run 6	1	0.1998	4351
Run 7	1.1	0.1757	3826
Run 8	1.2	0.1553	3382
Run 9	1.3	0.1381	3006
Run 10	1.4	0.1233	2686
Run 11	1.5	0.1107	2410
Run 12	1.6	0.09981	2173
Run 13	1.7	0.09036	1967
Run 14	1.8	0.08213	1788
Run 15	1.9	0.07492	1631
Run 16	2	0.06859	1493
Run 17	2.1	0.063	1372
Run 18	2.2	0.05804	1264
Run 19	2.3	0.05362	1167
Run 20	2.4	0.04968	1082
Run 21	2.5	0.04614	1005

Now, plot the results:
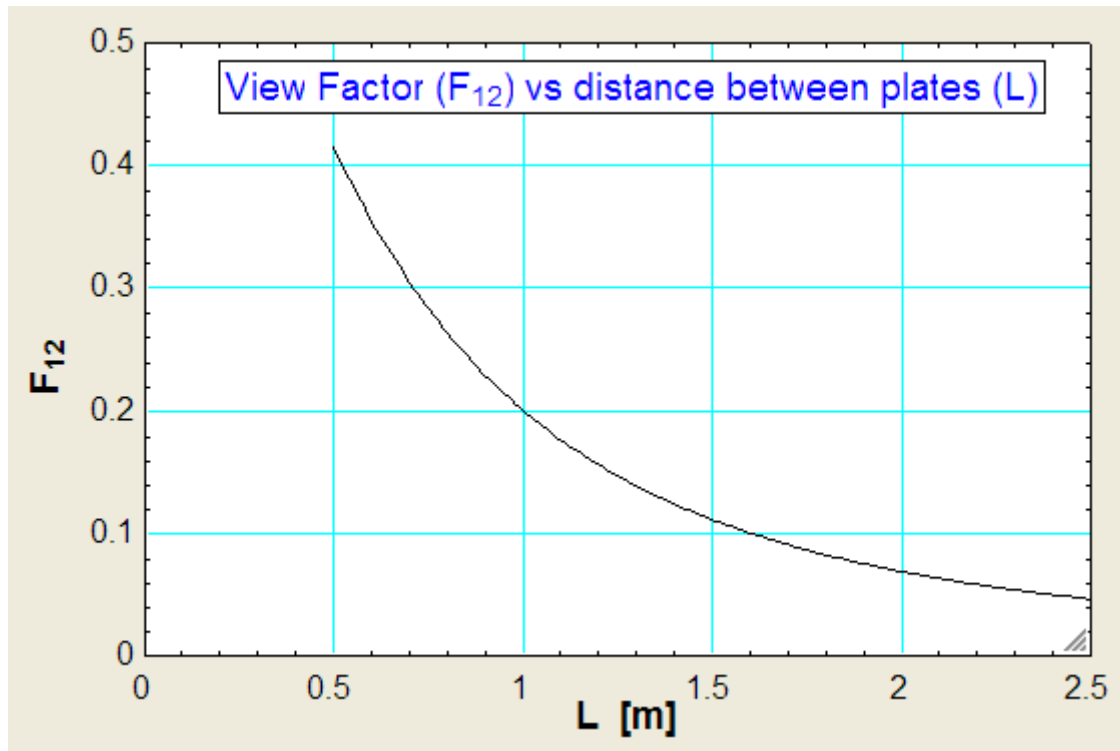
==
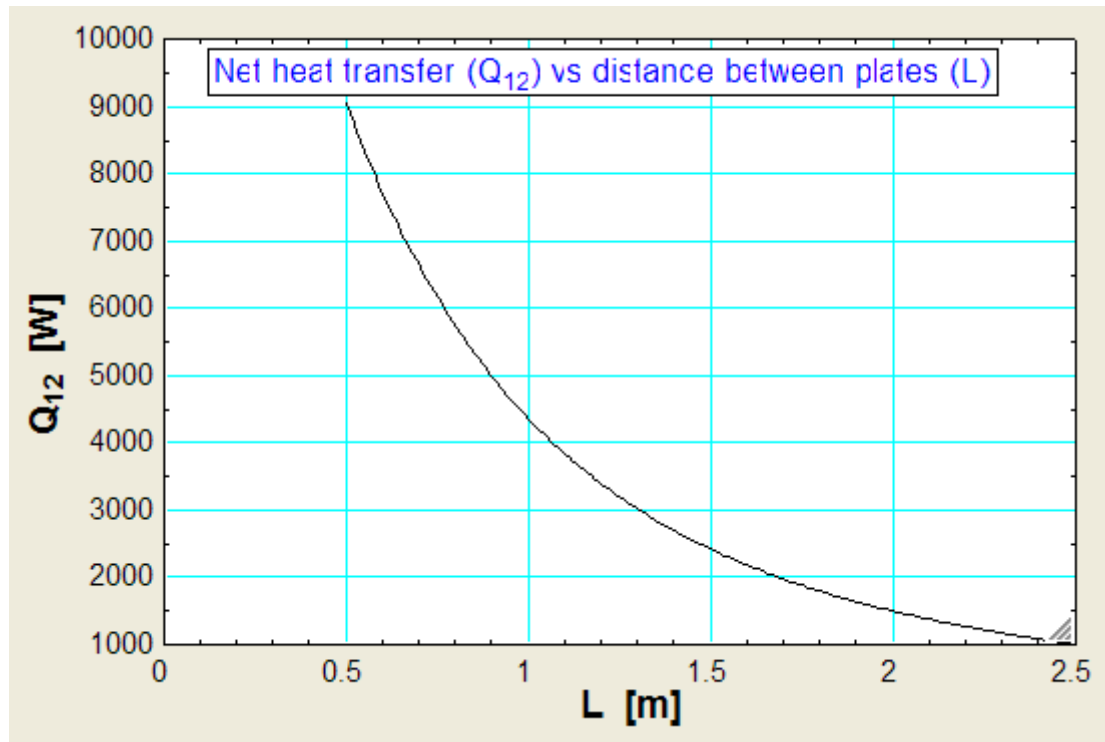
View Factors for two concentric cylinders of finite length:
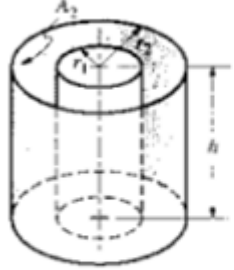
Let us write EES Functions for the view factors for:

a) **From the outer cylinder to itself,**
b) **From the outer cylinder to inner cylinder, and**
c) **From the inner cylinder to itself**

We use the view factor equations provided by Modest. (Ref: 'Radiative Heat Transfer' by Modest)

44 — Interior of finite-length, right-circular coaxial cylinder to itself

$$R = r_2/r_1, \quad H = h/r_1$$

$$F_{2-2} = 1 - \frac{1}{R} - \frac{\sqrt{H^2 + 4R^2} - H}{4R} + \frac{1}{\pi}\left(\frac{2}{R}\tan^{-1}\frac{2\sqrt{R^2 - 1}}{H}\right.$$

$$\left. - \frac{H}{2R}\left\{\frac{\sqrt{4R^2 + H^2}}{H}\sin^{-1}\frac{H^2 + 4(R^2 - 1) - 2H^2/R^2}{H^2 + 4(R^2 - 1)} - \sin^{-1}\frac{R^2 - 2}{R^2}\right\}\right)$$

45 — Interior of outer right-circular cylinder of finite length to exterior of inner right-circular coaxial cylinder

$$R = r_2/r_1, \quad H = h/r_1$$

$$F_{2-1} = \frac{1}{R}\left(1 - \frac{H^2 + R^2 - 1}{4H} - \frac{1}{\pi}\left\{\cos^{-1}\frac{H^2 - R^2 + 1}{H^2 + R^2 - 1}\right.\right.$$

$$\left.\left. - \frac{\sqrt{(H^2 + R^2 + 1)^2 - 4R^2}}{2H}\cos^{-1}\frac{H^2 - R^2 + 1}{R(H^2 + R^2 - 1)} - \frac{H^2 - R^2 + 1}{2H}\sin^{-1}\frac{1}{R}\right\}\right)$$

Note: In the EES Functions we write below, height of the cylinders (i.e. h in the above figs.) is taken as L.

FUNCTION F22_CoaxialCyl_outer_to_itself (R1, R2, L)

"Finds the View factor F_22 between two coaxial parallel cylinders – outer surface to itself"
"Ref: Radiative Heat Transfer by Modest"

R := R2/R1
H := L / R1
A = (sqrt(H ^ 2 + 4 * R ^ 2) – H) / (4 * R)
B = (2 * sqrt(R ^ 2 – 1)) / H
C = sqrt(4 * R ^ 2 + H ^ 2) / H

D = H ^ 2 + 4 * (R ^ 2 – 1) – 2 * H ^ 2 / R ^ 2

E = H ^ 2 + 4 * (R ^ 2 – 1)

F = (R ^ 2 – 2) / R ^ 2

G = (2 / R) * arctan(B)

F22_CoaxialCyl_outer_to_itself = 1 – 1 / R – A + (1 / pi) * (G – (H / (2 * R)) * (C *arcsin(D / E) – arcsin(F)))

END

"---"

FUNCTION F21_coaxialCyl_outer_to_inner(R1, R2, L)

"Finds the View factor F_21 between two coaxial parallel cylinders – outer surface to inner surface
Ref: Radiative Heat Transfer by Modest"

R := R2/R1

H := L / R1

A := (H ^ 2 + R ^ 2 – 1) / (4 * H)

B := (H ^ 2 – R ^ 2 + 1) / (H ^ 2 + R ^ 2 – 1)

C := sqrt((H ^ 2 + R ^ 2 + 1) ^ 2 – 4 * R ^ 2) / (2 * H)

D := (H ^ 2 – R ^ 2 + 1) / (R * (H ^ 2 + R ^ 2 – 1))

E := (H ^ 2 – R ^ 2 + 1) / (2 * H)

F21_coaxialCyl_outer_to_inner = (1 / R) * (1 – A – (1 / pi) * (arccos(B) – C * arccos(D) – E * arcsin(1 / R)))

END

"---"

FUNCTION F12_Coaxial_cyl_inner_to_outer (R1, R2, L)

{Finds the View factor F12 between two coaxial parallel cylinders – inner surface to outer surface. Ref: F_21 From Modest, see above.

Use: A2 * F_21 = A1 * F_12;

i.e. F_12 = F_21*(A2/A1);

And, A2/A1 = R2/R1}

F12_Coaxial_cyl_inner_to_outer = (R2/R1) * F21_coaxialCyl_outer_to_inner(R1, R2, L)

END

"---"

Now, let us use these Functions in solving a problem:

Prob.5B.10. Two concentric cylinders have diameters of 10 and 20 cm. Calculate the View factor between the two open ends.
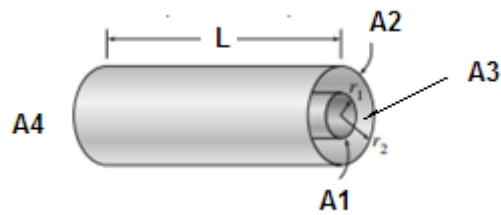
Fig.Prob.5B.10

EES Solution:

Let: Surface 1: outer surface of inner cylinder

Surface 2: inner surface of outer cylinder

Surfaces 3 and 4: surfaces at the two ends

Then, we have:

"Data:"

R1 = 0.05 [m]
R2 = 0.1 [m]
L = 0.2 [m]

"Calculations:"

A1 = 2 * pi * R1 * L "[m^2] … area of inner cylinder surface"

A2 = 2 * pi * R2 * L "[m^2] … area of outer cylinder surface"

A3 = pi * (R2^2 – R1^2) "[m^2] … area of open, end surface"

A4 = A3 "[m^2] … area of open, end surface"

F_21 = F21_CoaxialCyl_outer_to_inner(R1, R2, L) "…finds View factor from outer to inner surface"

F_22 = F22_CoaxialCyl_outer_to_itself (R1, R2, L) "…finds View factor from outer surface to itself"

"Now, by reciprocity:"

A1 * F_12 = A2 * F_21 "..finds F_12"

"For surface 2, Summation rule gives:"

F_21 + F_22 + F_23 + F_24 = 1

"But, by symmetry:"

F_23 = F_24 "…therefore, F_23 and F_24 are calculated."

"Again, applying reciprocity between surfaces 2 and 3:"

A2 * F_23 = A3 * F_32 "…finds F_32"

"Now,"

F_33 = 0 "…since surface is flat"

F_44 = 0 "…since surface is flat"

F_11 = 0 "…since surface is convex"

"Applying Summation rule for surface 3:"

$F\_31 + F\_32 + F\_33 + F\_34 = 1$ "....eq. (A)"

"In the above, F_31 and F_34 are unknown."

"To find F_31:"

$F\_11 + F\_12 + F\_13 + F\_14 = 1$ "…by Summation rule on surface 1"

"But,"

$F\_13 = F\_14$ "…by Symmetry"

"Therefore, F_13 and F_14 are determined, since F_11 = 0 and F_12 is already determined"

"Then, by reciprocity:"

$A1 * F\_13 = A3 * F\_31$ "…finds F_31"

"And, from eqn.(A), F_34 is immediately calculated."

Results:

Unit Settings: SI C Pa J mass rad

$A1 = 0.06283 \, [m^2]$	$A2 = 0.1257 \, [m^2]$	$A3 = 0.02356 \, [m^2]$	$A4 = 0.02356 \, [m^2]$
$F_{11} = 0$	$F_{12} = 0.8253$	$F_{13} = 0.08737$	$F_{14} = 0.08737$
$F_{21} = 0.4126$	$F_{22} = 0.3286$	$F_{23} = 0.1294$	$F_{24} = 0.1294$
$F_{31} = 0.233$	$F_{32} = 0.6901$	$F_{33} = 0$	$F_{34} = 0.07694$
$F_{44} = 0$	$L = 0.2 \, [m]$	$R1 = 0.05 \, [m]$	$R2 = 0.1 \, [m]$

Thus:

View factor between the two open ends = F_34 = 0.07694 … Ans.

Also, plot F_34 for different values of L, i.e. 0.1 < L < 0.5 m:

Parametric Table:

1..9	L [m]	F_{34}
Run 1	0.1	0.2036
Run 2	0.15	0.1194
Run 3	0.2	0.07694
Run 4	0.25	0.05309
Run 5	0.3	0.03858
Run 6	0.35	0.02918
Run 7	0.4	0.02279
Run 8	0.45	0.01826
Run 9	0.5	0.01494

Now, plot the results:

==
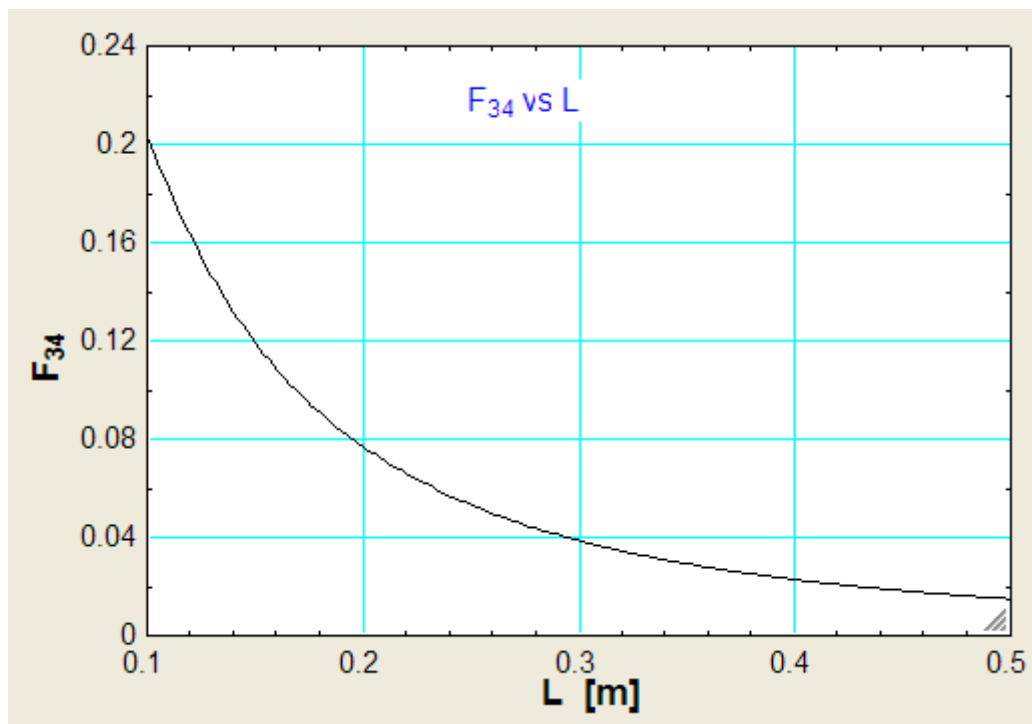
Now, let us write VBA Functions for view factors for a few important geometries:

We shall write VBA Functions for:

1. **Aligned, parallel rectangles,**
2. **Coaxial, parallel disks, and**
3. **Perpendicular rectangles with a common edge**

Recollect the nomenclature of geometries:
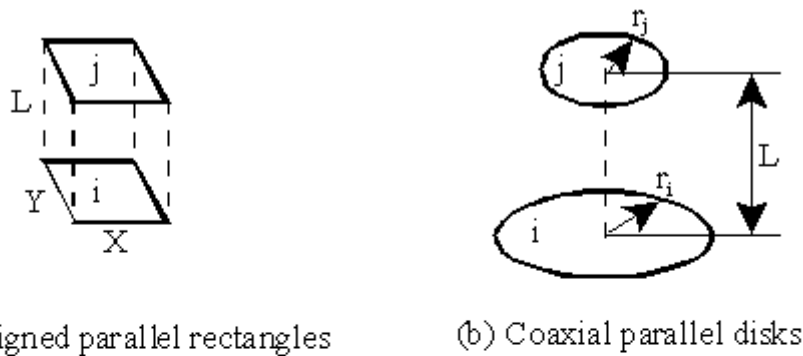
(a) Aligned parallel rectangles   (b) Coaxial parallel disks

(c) Perpendicular rectangles with a common edge

Fig.13.18 Few three-dimensional geometries

And, the equations for View factors are given at the beginning of the chapter.

Now, we shall present the VBA programs for View factors:

----

1. **Aligned, parallel rectangles:**

```
Option Explicit
Function F_ij_Aligned_parallel_rectangles(XBYL As Double, YBYL As Double) As Double
'Finds the View factor F_ij between two aligned parallel rectangles
'Note that VB does not have the ATAN function
'So, use the relation: ATAN(x) = ASIN(x/sqrt(x^2 +1))
Dim XX As Double, YY As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, F As Double, BB As Double, CC As Double
XX = XBYL
YY = YBYL
BB = (XX / (1 + YY ^ 2) ^ (1 / 2))
CC = (YY / (1 + XX ^ 2) ^ (1 / 2))
A = 2 / (Application.Pi() * XX * YY)
B = Application.Ln(Sqr((1 + XX ^ 2) * (1 + YY ^ 2) / (1 + XX ^ 2 + YY ^ 2)))
'C = XX * (1 + YY ^ 2) ^ (1 / 2) * Atan(XX / (1 + YY ^ 2) ^ (1 / 2))
'D = YY * (1 + XX ^ 2) ^ (1 / 2) * Atan(YY / (1 + XX ^ 2) ^ (1 / 2))
C = XX * (1 + YY ^ 2) ^ (1 / 2) * Application.Asin(BB / Sqr(BB ^ 2 + 1))
D = YY * (1 + XX ^ 2) ^ (1 / 2) * Application.Asin(CC / Sqr(CC ^ 2 + 1))
'E = XX * Atan(XX)
'F = YY * Atan(YY)
E = XX * Application.Asin(XX / Sqr(XX ^ 2 + 1))
F = YY * Application.Asin(YY / Sqr(YY ^ 2 + 1))
F_ij_Aligned_parallel_rectangles = A * (B + C + D - E - F)
End Function
```

2. **Coaxial, parallel disks:**

```
Function F_ij_Coaxial_parallel_discs(L_by_r_i As Double, r_j_by_L As Double) As Double
'Finds the View factor F_ij between two coaxial parallel discs
Dim RR_i As Double, RR_j As Double, S As Double
RR_i = 1 / L_by_r_i
RR_j = r_j_by_L
S = 1 + (1 + RR_j ^ 2) / RR_i ^ 2
F_ij_Coaxial_parallel_discs = (1 / 2) * (S - (S ^ 2 - 4 * (RR_j / RR_i) ^ 2) ^ 0.5)
End Function
```

3. **Perpendicular rectangles with a common edge:**

```
Function F_ij_Perpendicular_rectangles(ZBYX As Double, YBYX As Double) As Double
'Finds the View factor F_ij between two perpendicular rectangles with a common edge
'common edge: X; other sides:Y, Z
Dim H As Double, W As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, E_1 As Double, E_2 As Double, E_3 As Double
H = ZBYX
W = YBYX
A = 1 / (Application.Pi() * W)
B = W * Application.Asin((1 / W) / Sqr((1 / W) ^ 2 + 1))
C = H * Application.Asin((1 / H) / Sqr((1 / H) ^ 2 + 1))
D = (H ^ 2 + W ^ 2) ^ 0.5 * Application.Asin((1 / Sqr(H ^ 2 + W ^ 2)) / Sqr((1 / _
(H ^ 2 + W ^ 2)) + 1))
E_1 = (1 + W ^ 2) * (1 + H ^ 2) / (1 + W ^ 2 + H ^ 2)
E_2 = (W ^ 2 * (1 + W ^ 2 + H ^ 2) / ((1 + W ^ 2) * (W ^ 2 + H ^ 2))) ^ (W ^ 2)
E_3 = (H ^ 2 * (1 + H ^ 2 + W ^ 2) / ((1 + H ^ 2) * (H ^ 2 + W ^ 2))) ^ (H ^ 2)
E = E_1 * E_2 * E_3
F_ij_Perpendicular_rectangles = A * (B + C - D + 0.25 * Application.Ln(E))
End Function
```

Prob.5B.11. Draw the View factor graphs for Aligned, parallel rectangles:

EXCEL Solution:

We shall use the VBA Function written above.

Following are the steps:

1. Set up the EXCEL worksheet. Set up a Table as follows:

	D82			f_x	=F_ij_Aligned_parallel_rectangles(C82,$D$81)							
	A	B	C	D	E	F	G	H	I	J	K	
80												
81			XbyL\YbyL =	0.1	0.2	0.4	0.6	1	2	4	10	
82			0.1	0.003162057	0.00626	0.01207	0.01715	0.02492	0.03514	0.04209	0.04671
83			0.2									
84			0.3									
85			0.4									
86			0.5									
87			0.6									
88			0.7									
89			0.8									
90			0.9									
91			1									
92			2									
93			3									
94			4									
95			5									
96			6									
97			7									
98			8									
99			9									
100			10									
101			20									
102			30									
103			40									
104			50									
105			60									
106			70									
107			80									
108			90									
109			100									

In the above screen shot, column C contains X/L values and row 81 contains Y/L values.

We use the VBA Function written above to calculate the view factor F_{ij} (i.e. view factor from surface i to surface j), for given values of X/L and Y/L. Formula entered in cellD82 can be seen in the Formula bar. Use relative references to X/L so that we can drag-copy to fill up the Table later. Similarly, write formulas for other Y/L values as shown above.

Recollect that to enter the desired VBA Function, click on 'Insert Function' mark:
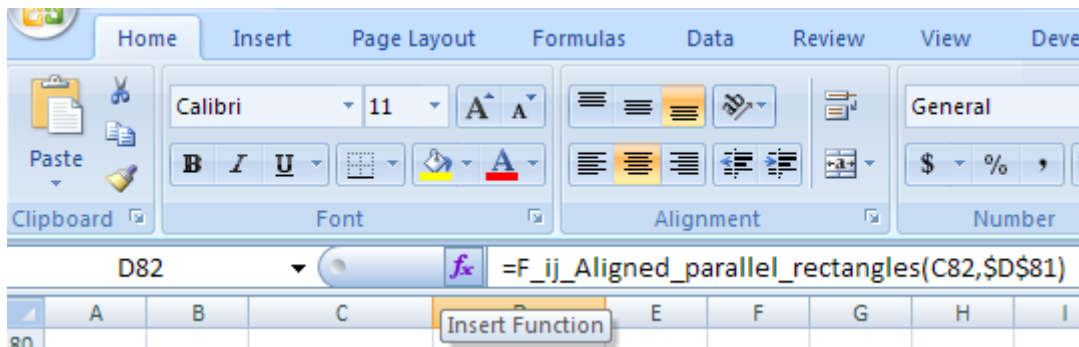
Clicking on 'Insert Function', gives:
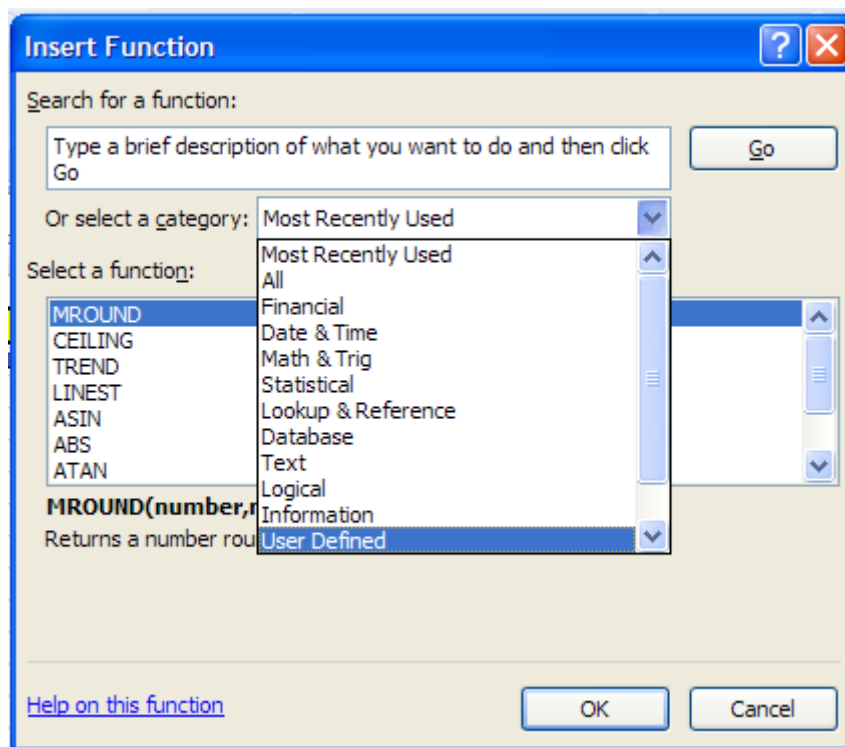
In the above window, click on 'User Defined' category and press OK:

Choose 'F_ij Aligned, parallel rectangles' and press OK. We get the following window; fill it up as shown. Note that cell C82 for X/L is in relative reference and cell D81 for Y/L is in absolute reference, so that when we drag-copy downwards, appropriate X/L values are automatically chosen:
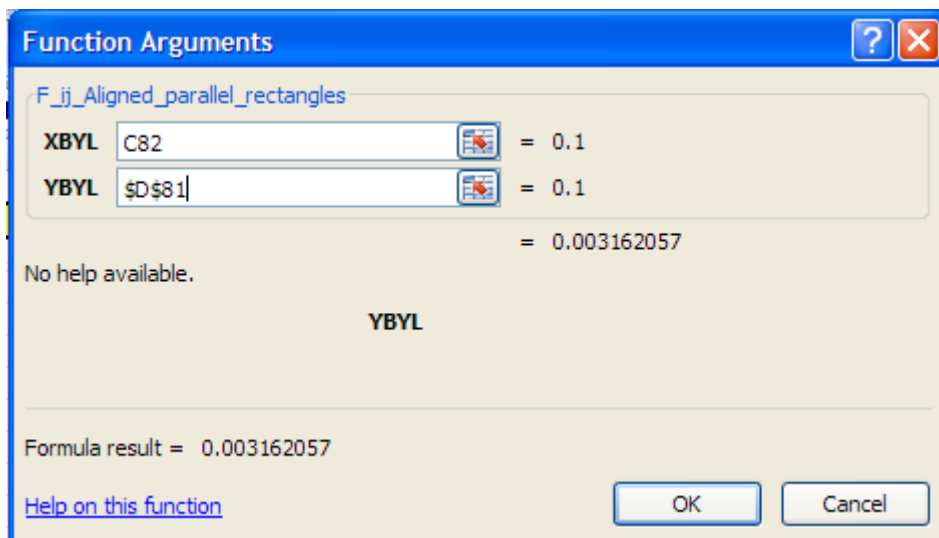
Click OK. And the value of F_ij is calculated.

Complete the row 82 likewise.

2. Now, select cells D82 to K82 and drag-copy to the end of the Table, i.e. up to the cell K109. The calculations are done immediately and the Table is filled up:

XbyL\YbyL =	0.1	0.2	0.4	0.6	1	2	4	10
0.1	0.0031621	0.00626	0.01207	0.01715	0.02492	0.03514	0.04209	0.04671
0.2	0.006263	0.0124	0.02392	0.03398	0.04941	0.06971	0.08353	0.09272
0.3	0.009247	0.01832	0.03532	0.0502	0.07305	0.10318	0.12372	0.13739
0.4	0.012073	0.02392	0.04614	0.0656	0.09554	0.13513	0.16219	0.18021
0.5	0.014711	0.02915	0.05625	0.08001	0.11665	0.16527	0.19859	0.22081
0.6	0.017148	0.03398	0.0656	0.09336	0.13627	0.19342	0.23271	0.25896
0.7	0.019379	0.0384	0.07417	0.10563	0.15435	0.21951	0.26447	0.29455
0.8	0.021411	0.04243	0.08199	0.11683	0.17092	0.24356	0.29387	0.32759
0.9	0.023255	0.04609	0.0891	0.12702	0.18605	0.26564	0.321	0.35816
1	0.024925	0.04941	0.09554	0.13627	0.19982	0.28588	0.34596	0.38638
2	0.035143	0.06971	0.13513	0.19342	0.28588	0.41525	0.50899	0.57338
3	0.039652	0.07868	0.15268	0.21888	0.32474	0.47558	0.58762	0.66599
4	0.042091	0.08353	0.16219	0.23271	0.34596	0.50899	0.63204	0.71933
5	0.043603	0.08654	0.16809	0.2413	0.35917	0.52993	0.6602	0.75363
6	0.044628	0.08858	0.17209	0.24712	0.36813	0.54421	0.67954	0.77741
7	0.045366	0.09005	0.17497	0.25132	0.37461	0.55454	0.69359	0.79482
8	0.045924	0.09116	0.17715	0.25449	0.3795	0.56236	0.70426	0.80811
9	0.046359	0.09202	0.17885	0.25697	0.38332	0.56847	0.71261	0.81856
10	0.046708	0.09272	0.18021	0.25896	0.38638	0.57338	0.71933	0.82699
20	0.048288	0.09586	0.18638	0.26795	0.40026	0.59563	0.7499	0.86563
30	0.048817	0.09691	0.18845	0.27096	0.40491	0.60309	0.76017	0.8787
40	0.049081	0.09744	0.18948	0.27246	0.40723	0.60682	0.76532	0.88526
50	0.049240	0.09775	0.1901	0.27337	0.40863	0.60906	0.76841	0.8892
60	0.049346	0.09797	0.19051	0.27397	0.40956	0.61056	0.77047	0.89183
70	0.049422	0.09812	0.19081	0.2744	0.41022	0.61162	0.77194	0.89371
80	0.049478	0.09823	0.19103	0.27472	0.41072	0.61243	0.77304	0.89512
90	0.049523	0.09832	0.1912	0.27497	0.41111	0.61305	0.7739	0.89622
100	0.049558	0.09839	0.19134	0.27517	0.41142	0.61355	0.77459	0.89709

3. Now draw the graphs in EXCEL:

===
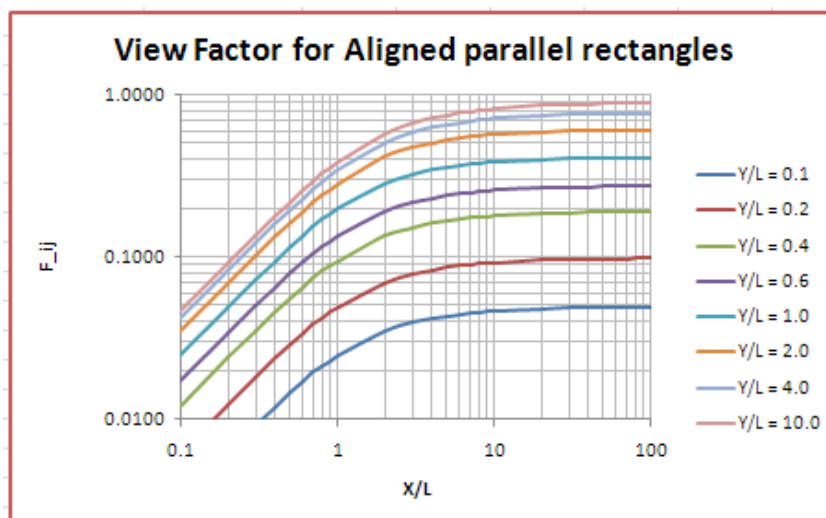
Prob.5B.12. Draw the View factor graphs for coaxial, parallel disks:

EXCEL Solution:

We shall use the VBA Function written above.

Following are the steps:

1. Set up the EXCEL worksheet. Set up a Table as follows:

| D141		f_x	=F_ij_Coaxial_parallel_discs(C141,$D$140)						
	A	B	C	D	E	F	G	H	I	J
139										
140			L_byR_i\R_jby_L	0.3	0.6	1	1.5	2	4	8
141			0.1	0.000891081	0.00356	0.0099	0.02227	0.03959	0.15812	0.62344
142			0.2							
143			0.3							
144			0.4							
145			0.5							
146			0.6							
147			0.7							
148			0.8							
149			0.9							
150			1							
151			2							
152			3							
153			4							
154			5							
155			6							
156			7							
157			8							
158			9							
159			10							
160			20							
161			30							
162			40							
163			50							
164			60							
165			70							
166			80							
167			90							
168			100							

In the above screen shot, column C contains L/R_i values and row 140 contains R_j/L values.

We use the VBA Function written above to calculate the view factor F_ij (i.e. view factor from surface i to surface j), for given values of L/R_i and R_j/L. Formula entered in cell D141 can be seen in the Formula bar. Use relative references to L/R_i so that we can drag-copy to fill up the Table later. Similarly, write formulas for other R_j/L values as shown above.
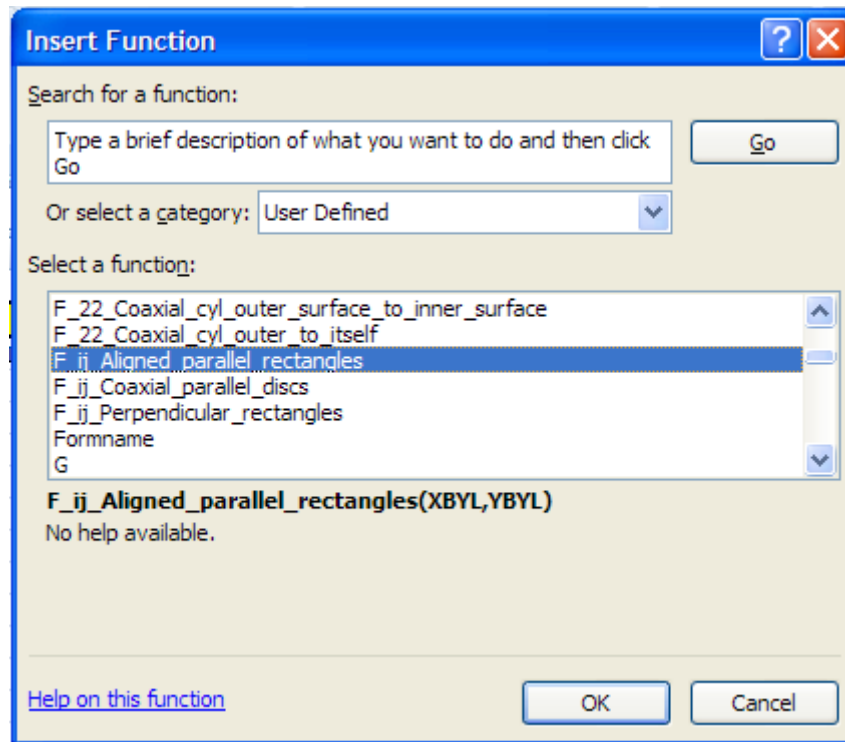
2. Now, select cells D141 to J141 and drag-copy to the end of the Table, i.e. up to the cell J168. The calculations are done immediately and the Table is filled up:

L_byR_i\R_jby_L	0.3	0.6	1	1.5	2	4	8
0.1	0.000891081	0.00356	0.0099	0.02227	0.03959	0.15812	0.62344
0.2	0.003461076	0.01384	0.0384	0.08623	0.15279	0.58388	0.97538
0.3	0.007426605	0.02965	0.08196	0.18242	0.31803	0.86472	0.98151
0.4	0.012392345	0.0493	0.13502	0.29352	0.48769	0.91153	0.98301
0.5	0.017934496	0.07092	0.19098	0.39754	0.60961	0.92482	0.98362
0.6	0.023671607	0.09278	0.24388	0.47901	0.67889	0.93063	0.98394
0.7	0.029306344	0.1136	0.28993	0.53614	0.71722	0.93375	0.98412
0.8	0.03463697	0.13258	0.32784	0.57481	0.73985	0.93564	0.98424
0.9	0.039547509	0.14937	0.35811	0.60123	0.75423	0.93688	0.98432
1	0.043988012	0.16393	0.38197	0.6198	0.76393	0.93774	0.98438
2	0.068027565	0.23196	0.46887	0.67544	0.79176	0.94035	0.98456
3	0.075457337	0.24941	0.48612	0.68493	0.7964	0.94081	0.98459
4	0.078424643	0.25595	0.49219	0.68818	0.79798	0.94097	0.9846
5	0.079871841	0.25906	0.495	0.68967	0.79871	0.94105	0.98461
6	0.080678654	0.26077	0.49653	0.69048	0.79911	0.94109	0.98461
7	0.081172376	0.26181	0.49745	0.69097	0.79935	0.94111	0.98461
8	0.081495782	0.26248	0.49805	0.69128	0.7995	0.94113	0.98461
9	0.081718871	0.26295	0.49846	0.6915	0.7996	0.94114	0.98461
10	0.081879129	0.26328	0.49875	0.69165	0.79968	0.94114	0.98461
20	0.082395398	0.26435	0.49969	0.69214	0.79992	0.94117	0.98461
30	0.082491655	0.26455	0.49986	0.69223	0.79996	0.94117	0.98462
40	0.082525393	0.26462	0.49992	0.69227	0.79998	0.94117	0.98462
50	0.082541017	0.26465	0.49995	0.69228	0.79999	0.94118	0.98462
60	0.082549507	0.26467	0.49997	0.69229	0.79999	0.94118	0.98462
70	0.082554627	0.26468	0.49997	0.69229	0.79999	0.94118	0.98462
80	0.08255795	0.26468	0.49998	0.6923	0.79999	0.94118	0.98462
90	0.082560228	0.26469	0.49998	0.6923	0.8	0.94118	0.98462
100	0.082561858	0.26469	0.49999	0.6923	0.8	0.94118	0.98462

3. Now, draw the graphs in EXCEL:

===

Prob.5B.13. Draw the View factor graphs for Perpendicular rectangles with a common edge:

EXCEL Solution:

We shall use the VBA Function written above.

Following are the steps:

1. Set up the EXCEL worksheet. Set up a Table as follows:

D207			f_x	=F_ij_Perpendicular_rectangles(C207,$D$206)							
	A	B	C	D	E	F	G	H	I	J	K	L
205												
206			ZbyX\YbyX =	0.02	0.05	0.1	0.2	0.6	1	2	4	10
207			0.1	0.443749568	0.37202	0.28189	0.18108	0.07048	0.04325	0.02188	0.01097	0.00439
208			0.2									
209			0.3									
210			0.4									
211			0.5									
212			0.6									
213			0.7									
214			0.8									
215			0.9									
216			1									
217			1.5									
218			2									
219			2.5									
220			3									
221			3.5									
222			4									
223			4.5									
224			5									
225			5.5									
226			6									
227			6.5									
228			7									
229			7.5									
230			8									
231			8.5									
232			9									
233			9.5									
234			10									

In the above screen shot, column C contains Z/X values and row 206 contains Y/X values.

We use the VBA Function written above to calculate the view factor F_ij (i.e. view factor from surface i to surface j), for given values of Z/X and Y/X. Formula entered in cell D207 can be seen in the Formula bar. Use relative references to Z/X so that we can drag-copy to fill up the Table later. Similarly, write formulas for other Y/X values as shown above.

2. Now, select cells D207 to L207 and drag-copy to the end of the Table, i.e. up to the cell L234. The calculations are done immediately and the Table is filled up:

ZbyX\YbyX =	0.02	0.05	0.1	0.2	0.6	1	2	4	10
0.1	0.443749568	0.37202	0.28189	0.18108	0.07048	0.04325	0.02188	0.01097	0.00439
0.2	0.466154321	0.42337	0.36216	0.27104	0.12295	0.07744	0.03971	0.02	0.00802
0.3	0.473183243	0.44044	0.39322	0.31714	0.16239	0.10512	0.05473	0.02768	0.01111
0.4	0.476459637	0.44852	0.40859	0.34295	0.19206	0.1277	0.06757	0.03434	0.0138
0.5	0.478288879	0.45305	0.41738	0.35864	0.21445	0.14619	0.07865	0.04018	0.01617
0.6	0.479423474	0.45587	0.4229	0.36884	0.23147	0.16138	0.08829	0.04536	0.01829
0.7	0.480178071	0.45775	0.4266	0.37581	0.24453	0.1739	0.09671	0.04998	0.02019
0.8	0.480705777	0.45906	0.4292	0.38078	0.25467	0.18425	0.10411	0.05413	0.02191
0.9	0.481089185	0.46002	0.43109	0.38443	0.26263	0.19286	0.11065	0.05789	0.02348
1	0.481376318	0.46073	0.43251	0.38719	0.26896	0.20004	0.11643	0.06131	0.02492
1.5	0.482110417	0.46257	0.43616	0.39435	0.28678	0.22232	0.13715	0.07458	0.03069
2	0.482390134	0.46327	0.43756	0.39711	0.29429	0.23285	0.1493	0.08365	0.03491
2.5	0.482524639	0.4636	0.43823	0.39844	0.29805	0.23847	0.15682	0.09015	0.03818
3	0.482599192	0.46379	0.4386	0.39919	0.30019	0.24176	0.16169	0.09495	0.04082
3.5	0.482644682	0.4639	0.43883	0.39964	0.30151	0.24383	0.16499	0.09857	0.043
4	0.48267443	0.46398	0.43897	0.39993	0.30238	0.24522	0.16731	0.10136	0.04484
4.5	0.48269493	0.46403	0.43908	0.40014	0.30298	0.24619	0.16899	0.10353	0.04641
5	0.482709646	0.46406	0.43915	0.40029	0.30342	0.2469	0.17024	0.10526	0.04777
5.5	0.482720563	0.46409	0.43921	0.4004	0.30374	0.24743	0.17119	0.10664	0.04895
6	0.482728883	0.46411	0.43925	0.40048	0.30399	0.24783	0.17193	0.10776	0.04998
6.5	0.482735367	0.46413	0.43928	0.40054	0.30418	0.24815	0.17252	0.10868	0.05089
7	0.482740519	0.46414	0.4393	0.40059	0.30433	0.2484	0.17299	0.10944	0.0517
7.5	0.482744679	0.46415	0.43933	0.40064	0.30446	0.2486	0.17338	0.11008	0.05242
8	0.482748086	0.46416	0.43934	0.40067	0.30456	0.24877	0.1737	0.11062	0.05306
8.5	0.482750912	0.46417	0.43936	0.4007	0.30464	0.24891	0.17397	0.11108	0.05363
9	0.482753281	0.46417	0.43937	0.40072	0.30471	0.24903	0.17419	0.11147	0.05414
9.5	0.482755287	0.46418	0.43938	0.40074	0.30477	0.24912	0.17438	0.11181	0.05461
10	0.482757001	0.46418	0.43939	0.40076	0.30482	0.24921	0.17455	0.1121	0.05502

1. Now, draw the graphs in EXCEL:

===
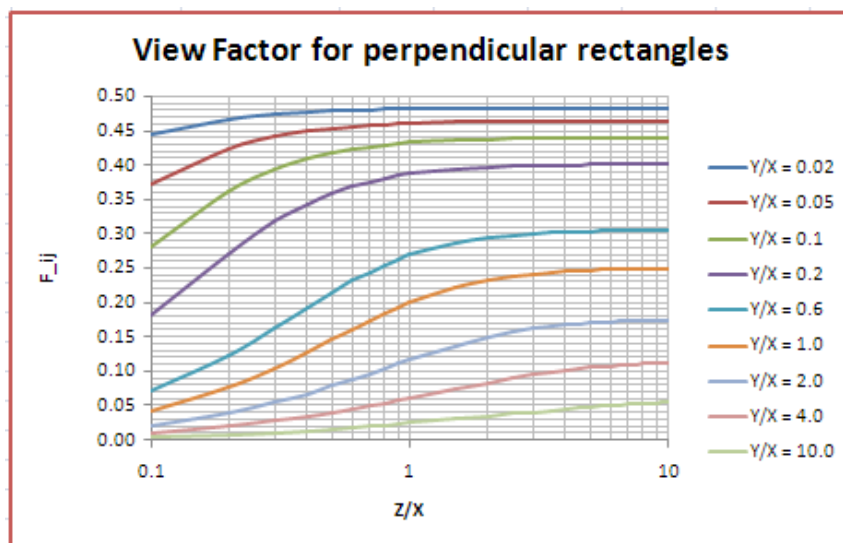
Prob.5B.14. Draw the View factor graphs for two concentric cylinders of finite length:

Recollect:

View Factors for two concentric cylinders of finite length:

Let us write EES Functions for the view factors for:

a) **From the outer cylinder to inner cylinder, F21,**

b) **From the inner cylinder to outer cylinder, F12**

c) **From outer cylinder to itself, F22**

We use the view factor equations provided by Modest. (Ref: 'Radiative Heat Transfer' by Modest)

44   Interior of finite-length, right-circular coaxial cylinder to itself

$$R = r_2/r_1, \quad H = h/r_1$$

$$F_{2-2} = 1 - \frac{1}{R} - \frac{\sqrt{H^2 + 4R^2} - H}{4R} + \frac{1}{\pi}\left(\frac{2}{R}\tan^{-1}\frac{2\sqrt{R^2-1}}{H}\right.$$
$$\left. - \frac{H}{2R}\left\{\frac{\sqrt{4R^2 + H^2}}{H}\sin^{-1}\frac{H^2 + 4(R^2-1) - 2H^2/R^2}{H^2 + 4(R^2-1)} - \sin^{-1}\frac{R^2-2}{R^2}\right\}\right)$$

45   Interior of outer right-circular cylinder of finite length to exterior of inner right-circular coaxial cylinder

$$R = r_2/r_1, \quad H = h/r_1$$

$$F_{2-1} = \frac{1}{R}\left(1 - \frac{H^2 + R^2 - 1}{4H} - \frac{1}{\pi}\left\{\cos^{-1}\frac{H^2 - R^2 + 1}{H^2 + R^2 - 1}\right.\right.$$
$$\left.\left. - \frac{\sqrt{(H^2 + R^2 + 1)^2 - 4R^2}}{2H}\cos^{-1}\frac{H^2 - R^2 + 1}{R(H^2 + R^2 - 1)} - \frac{H^2 - R^2 + 1}{2H}\sin^{-1}\frac{1}{R}\right\}\right)$$

First, let us write VBA Functions for the View factors, using the above equations:

1. **View Factor F_21 from outer cylinder to inner cylinder for different L/R2:**

```
Function F_21_Coaxial_cyl_outer_surface_to_inner_surface(R1_by_R2 As Double, _
H_by_R2 As Double) As Double

'Finds the View factor F_21 between two coaxial parallel cylinders--outer surface to
'inner surface
'Ref: Radiative Heat Transfer by Modest
Dim R As Double, H As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double
R = 1 / R1_by_R2
H = H_by_R2 * R
A = (H ^ 2 + R ^ 2 - 1#) / (4# * H)
B = (H ^ 2 - R ^ 2 + 1#) / (H ^ 2 + R ^ 2 - 1#)
C = Sqr((H ^ 2 + R ^ 2 + 1) ^ 2 - 4 * R ^ 2) / (2# * H)
D = (H ^ 2 - R ^ 2 + 1#) / (R * (H ^ 2 + R ^ 2 - 1#))
E = (H ^ 2 - R ^ 2 + 1#) / (2# * H)
F_21_Coaxial_cyl_outer_surface_to_inner_surface = (1# / R) * (1# - A - (1# / Application.Pi()) * _
(Application.Acos(B) - C * Application.Acos(D) - E * Application.Asin(1 / R)))
End Function
```

2. **View Factor F_12 from inner cylinder to outer cylinder for different L/R2:**

```
Function F_12_Coaxial_cyl_inner_surface_to_outer_surface(R1_by_R2 As Double, _
H_by_R2 As Double) As Double
'Ref: F_21 From Modest, see above
'Use: A2.F_21 = A1*F_12; i.e. F_12 = F_21*(A2/A1); And, A2/A1 = R2/R1
    F_12_Coaxial_cyl_inner_surface_to_outer_surface = _
    F_21_Coaxial_cyl_outer_surface_to_inner_surface(R1_by_R2, H_by_R2) * (1# / R1_by_R2)
End Function
```

3. **View Factor F_22 from outer cylinder to itself for different L/R2:**

```
Function F_22_Coaxial_cyl_outer_to_itself(R1_by_R2 As Double, H_by_R2 As Double) As Double
'Finds the View factor F_22 between two coaxial parallel cylinders--outer surface to itself
'Ref: Radiative Heat Transfer by Modest

Dim R As Double, H As Double, A As Double, B As Double, C As Double
Dim D As Double, E As Double, F As Double, G As Double

R = 1 / R1_by_R2
H = H_by_R2 * R

A = (Sqr(H ^ 2 + 4 * R ^ 2) - H) / (4 * R)
B = (2 * Sqr(R ^ 2 - 1)) / H
C = Sqr(4 * R ^ 2 + H ^ 2) / H
D = H ^ 2 + 4 * (R ^ 2 - 1) - 2 * H ^ 2 / R ^ 2
E = H ^ 2 + 4 * (R ^ 2 - 1)
F = (R ^ 2 - 2) / R ^ 2
'Note that VB does not have the ATAN function
'So, use the relation: ATAN(x) = ASIN(x/sqrt(x^2 +1))
'G = (2 / R) * Application.Atan(B)
G = (2 / R) * Application.Asin(B / Sqr(B ^ 2 + 1))

F_22_Coaxial_cyl_outer_to_itself = 1 - 1 / R - A + (1 / Application.Pi()) * (G - (H / (2 * R)) * _
(C * Application.Asin(D / E) - Application.Asin(F)))

End Function
```

Now, using the above Functions, let us draw the graphs for different View factors for two concentric cylinders of finite length:

Prob.5B.14. Draw the View factor graph for F_21 for two concentric cylinders of finite length:

EXCEL Solution:

1. Set up a Table in EXCEL worksheet as shown below:

| | E382 | | f_x | =F_21_Coaxial_cyl_outer_surface_to_inner_surface(D382,$E$38 |

	C	D	E	F	G	H	I	J
380								
381		(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	1.00E+06
382		0.001	6.35E-05	0.000156	0.000295	0.0005	0.000705	0.001
383		0.1						
384		0.15						
385		0.2						
386		0.25						
387		0.3						
388		0.35						
389		0.4						
390		0.45						
391		0.5						
392		0.55						
393		0.6						
394		0.65						
395		0.7						
396		0.75						
397		0.8						
398		0.85						
399		0.9						
400		0.95						
401		1						

In the above screen shot, column D contains R1/R2 values and row 381 contains L/R2 values.

We use the VBA Function written above to calculate the view factor F_21 (i.e. view factor from surface 2 to surface 1), for given values of R1/R2 and L/R2. Formula entered in cell E382 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.

2. Now, select cells E382 to J382 and drag-copy to the end of the Table, i.e. up to the cell J401. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	1.00E+06
0.001	6.35E-05	0.000156	0.000295	0.0005	0.000705	0.001
0.1	0.006898	0.016903	0.031709	0.052666	0.072561	0.1
0.15	0.010836	0.026504	0.04945	0.081196	0.110487	0.15
0.2	0.015184	0.037054	0.0687	0.111382	0.149592	0.2
0.25	0.020024	0.048732	0.089691	0.143382	0.189938	0.25
0.3	0.02546	0.061759	0.112694	0.177367	0.231589	0.3
0.35	0.031626	0.076413	0.138032	0.213524	0.274613	0.35
0.4	0.038702	0.093054	0.166088	0.252053	0.319078	0.4
0.45	0.046929	0.112143	0.197321	0.29317	0.365058	0.45
0.5	0.056639	0.134292	0.232274	0.337106	0.412628	0.5
0.55	0.068305	0.160317	0.271598	0.384111	0.461868	0.55
0.6	0.082621	0.191324	0.316063	0.434452	0.512864	0.6
0.65	0.100646	0.228841	0.366573	0.488412	0.565709	0.65
0.7	0.124079	0.274996	0.424187	0.546297	0.620505	0.7
0.75	0.155797	0.332781	0.490118	0.608437	0.677368	0.75
0.8	0.201036	0.406394	0.565734	0.675197	0.736435	0.8
0.85	0.270142	0.501603	0.652549	0.746993	0.797878	0.85
0.9	0.385426	0.625941	0.752238	0.824349	0.86193	0.9
0.95	0.59705	0.788409	0.86678	0.908035	0.928971	0.95
1	1	1	1	1	1	1

3. Now, draw the graphs in EXCEL:

==
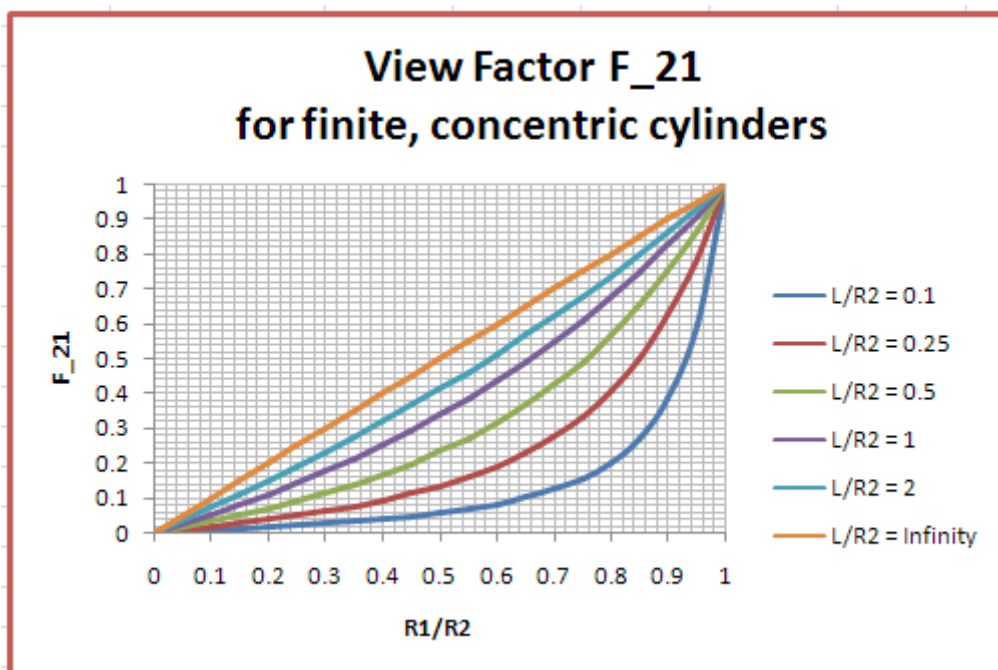
Prob.5B.15. Draw the View factor graph for F_12 for two concentric cylinders of finite length:

EXCEL Solution:

1. Set up a Table in EXCEL worksheet as shown below:

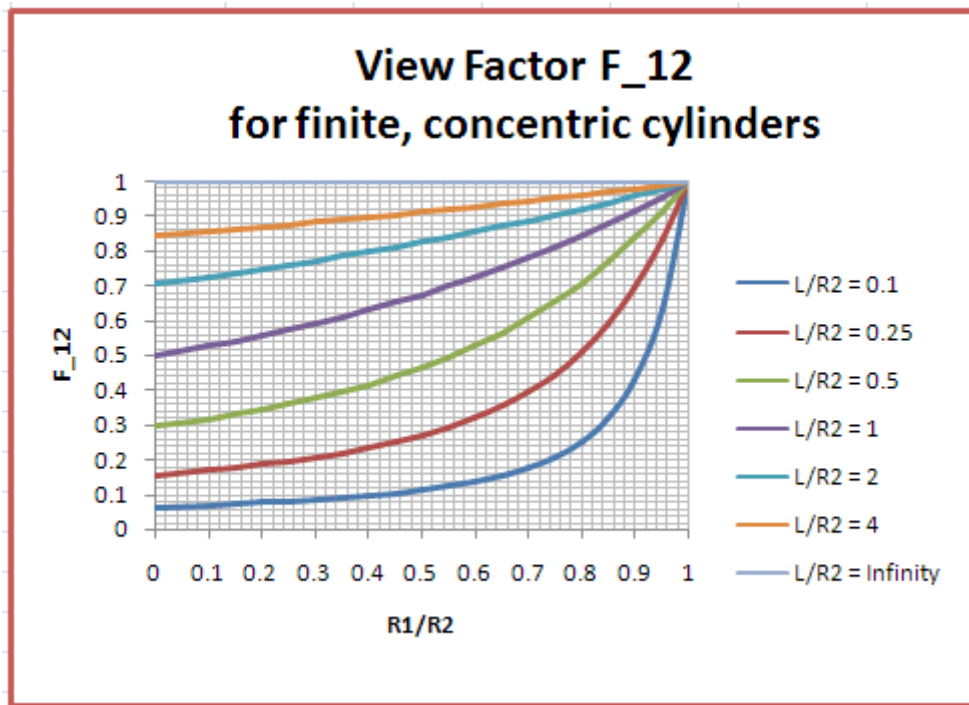|		E433		f_x	=F_22_Coaxial_cyl_outer_to_itself(D433,$E$432)					
	A	B	C	D	E	F	G	H	I	J	K
431											
432				(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
433				0.001	0.048719	0.117139	0.219068	0.381671	0.585286	0.763227	0.999
434				0.1							
435				0.15							
436				0.2							
437				0.25							
438				0.3							
439				0.35							
440				0.4							
441				0.45							
442				0.5							
443				0.55							
444				0.6							
445				0.65							
446				0.7							
447				0.75							
448				0.8							
449				0.85							
450				0.9							
451				0.95							
452				1							

In the above screen shot, column D contains R1/R2 values and row 483 contains L/R2 values.

We use the VBA Function written above to calculate the view factor F_12 (i.e. view factor from surface 1 to surface 2), for given values of R1/R2 and L/R2. Formula entered in cell E484 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.

2. Now, select cells E484 to K484 and drag-copy to the end of the Table, i.e. up to the cell K503. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
0.001	0.063501	0.156076	0.295367	0.50025	0.705033	0.844159	1
0.1	0.068977	0.169028	0.317088	0.526657	0.725605	0.856116	1
0.15	0.072242	0.176691	0.329665	0.541305	0.736581	0.862386	1
0.2	0.075922	0.185272	0.343501	0.556909	0.74796	0.868814	1
0.25	0.080096	0.194929	0.358764	0.573527	0.759752	0.875401	1
0.3	0.084865	0.205863	0.375646	0.591225	0.771964	0.882152	1
0.35	0.09036	0.218324	0.394376	0.610069	0.784608	0.889069	1
0.4	0.096756	0.232634	0.415221	0.630133	0.797696	0.896156	1
0.45	0.104287	0.249207	0.438491	0.651488	0.81124	0.903421	1
0.5	0.113279	0.268584	0.464549	0.674212	0.825256	0.910868	1
0.55	0.124191	0.291485	0.493815	0.698384	0.83976	0.918508	1
0.6	0.137702	0.318874	0.526771	0.724086	0.854773	0.92635	1
0.65	0.154841	0.352063	0.563959	0.751403	0.870321	0.934408	1
0.7	0.177256	0.392851	0.605982	0.780424	0.886435	0.942699	1
0.75	0.20773	0.443708	0.653491	0.81125	0.903157	0.951246	1
0.8	0.251295	0.507992	0.707167	0.843996	0.920544	0.960079	1
0.85	0.317814	0.590121	0.767705	0.878816	0.93868	0.969245	1
0.9	0.428251	0.69549	0.83582	0.915943	0.9577	0.978815	1
0.95	0.628474	0.829904	0.9124	0.955826	0.977864	0.988926	1
1	1	1	1	1	1	1	1

3. Now, draw the graphs in EXCEL:

===

Prob.5B.16. Draw the View factor graph for F_22 for two concentric cylinders of finite length:

EXCEL Solution:

1. Set up a Table in EXCEL worksheet as shown below:

In the above screen shot, column D contains R1/R2 values and row 432 contains L/R2 values.

We use the VBA Function written above to calculate the view factor F_22 (i.e. view factor from surface 2 to itself), for given values of R1/R2 and L/R2. Formula entered in cell E433 can be seen in the Formula bar. Use relative references to R1/R2 so that we can drag-copy to fill up the Table later. Similarly, write formulas for other L/R2 values as shown above.

2. Now, select cells E433 to K433 and drag-copy to the end of the Table, i.e. up to the cell
    K452. The calculations are done immediately and the Table is filled up:

(R1_by_R2)\L/R2	0.1	0.25	0.5	1	2	4	1.00E+07
0.001	0.04872	0.11714	0.21907	0.38167	0.58529	0.76323	0.999
0.1	0.04557	0.10929	0.2036	0.35241	0.53573	0.69341	0.9
0.15	0.04396	0.1053	0.19574	0.33755	0.51061	0.65806	0.85
0.2	0.04235	0.10128	0.18783	0.32259	0.48536	0.62262	0.8
0.25	0.04071	0.09722	0.17983	0.30749	0.45994	0.58705	0.75
0.3	0.03906	0.0931	0.17173	0.29223	0.43431	0.55131	0.7
0.35	0.03738	0.08892	0.16351	0.27674	0.40842	0.51537	0.65
0.4	0.03566	0.08465	0.15512	0.261	0.38222	0.47919	0.6
0.45	0.03391	0.08028	0.14654	0.24493	0.35563	0.44271	0.55
0.5	0.0321	0.07579	0.13773	0.22848	0.3286	0.4059	0.5
0.55	0.03023	0.07114	0.12863	0.21157	0.30104	0.36871	0.45
0.6	0.02829	0.06632	0.11919	0.1941	0.27286	0.33107	0.4
0.65	0.02625	0.06126	0.10932	0.17596	0.24395	0.29291	0.35
0.7	0.0241	0.05591	0.09891	0.157	0.21418	0.25415	0.3
0.75	0.02179	0.05019	0.08782	0.137	0.18337	0.21471	0.25
0.8	0.01927	0.04397	0.07582	0.11567	0.1513	0.17444	0.2
0.85	0.01645	0.03703	0.06253	0.09259	0.11765	0.1332	0.15
0.9	0.01316	0.02895	0.0473	0.06702	0.08197	0.09074	0.1
0.95	0.00894	0.0187	0.02861	0.03758	0.04347	0.04669	0.05
1	0	0	0	5.6E-17	-2.8E-17	4.2E-17	2.9E-12

3. Now, draw the graphs in EXCEL:

View Factor F_22
for finite, concentric cylinders

==
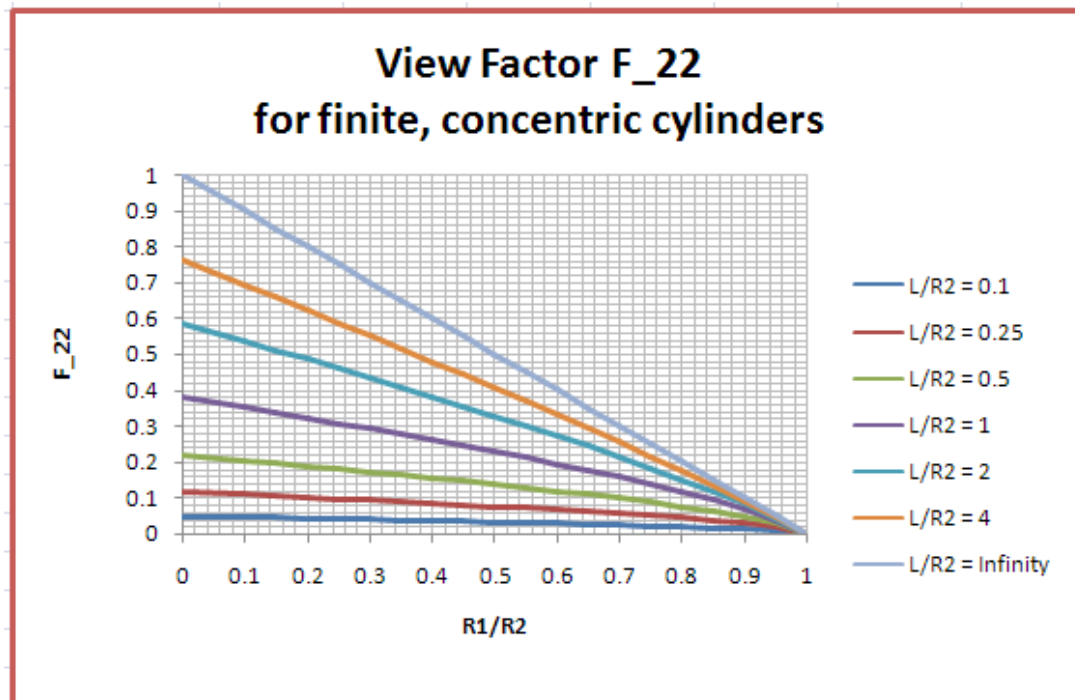
Prob.5B.17. Consider a hemispherical oven of 1 m diameter. Find the View factor of curved surface to itself. (= F22). The curved surface is at 800 K and the base is at 300 K. If both the surfaces can be considered as black, determine the heat transfer rate per unit length. Assume the furnace to be very long.
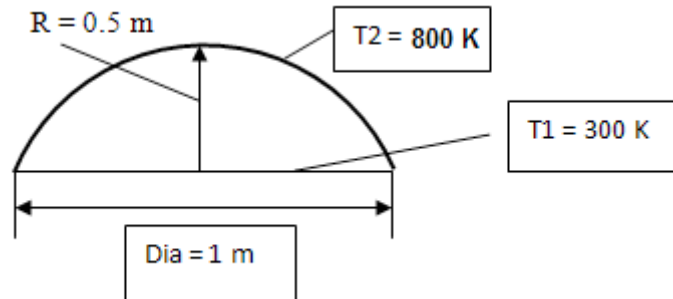

Fig.Prob.5B.17

EXCEL Solution:

Since the furnace is very long, there is no heat loss at the ends.

Excel worksheet is shown below. Formulas used are also shown in the worksheet.

C	D	E	F	G	H	I
Data:						
	D	1	m			
	T1	300	K			
	T2	800	K			
	L	1	m			
	sigma	5.67E-08	W/m^2.K^4	.....Stefan-Boltzmann const.		
Calculations:				A1 = D * L		
	A1	1	m^2			
	A2	1.57080	m^2	A2 = PI * (D/2) * L		
For Surface 1:	F11 + F12 = 1	..by Summation rule				
	But, F11 = 0 ....since surface is flat					
Therefore:	F12 =	1				
Then, by reciprocity:	A1 * F12 = A2 * F21					
Therefore:	F21 = (A1 / A2) * F12					
i.e.	F21 =	0.63662				
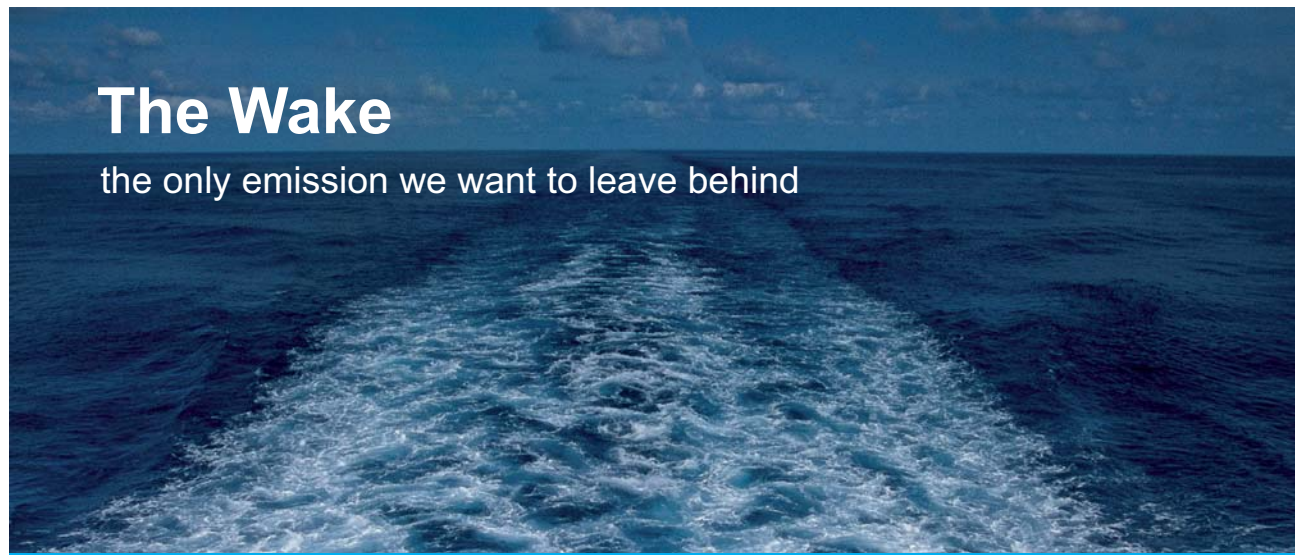
For Surface 2:	F21 + F22 = 1	..by Summation rule	
Then:	F22 =	0.36338	...Ans.
Heat Transferred:	Q12 = A2 * F21 * sigma * (T2^4 - T1^4)		
Then:	Q12 =	22765.05	W....Ans.

Thus:

F22 = View factor of curved surface to itself = 0.36338 … Ans.

Q12 = Net heat transfer between surface 2 and 1 = 22765 W…Ans.

==

Prob.5B.18. Consider a thin, hollow cylinder of 10 cm dia and 10 cm length, as shown in Fig. below.

a) What is the view factor of the curved surface to itself (i.e. F_{33})?
b) If T1 = 800 K, T2 = 400 K, find the net heat transfer between surfaces 1 and 2 (Q_{12}), considering all surfaces as black.
c) Also, plot the variation of F_{33} and Q_{12} as L varies from 0.1 m to 0.5 m
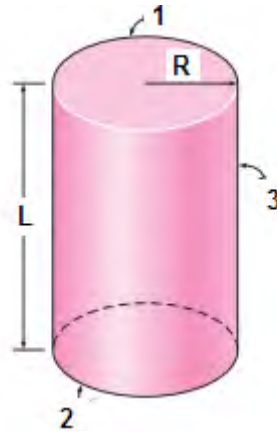
Fig.Prob.5B.18

EXCEL Solution:

Top, bottom and side surfaces are designated as 1, 2 and 3 respectively.

Let the radius be R and length, L.

Excel worksheet is shown below. Formulas used are also shown in the worksheet.

Following are the steps:

1. Set up the EXCEL worksheet, enter data:

	A	B	C	D	E	F
565			Data:			
566				R	0.05	m
567				T1	800	K
568				T2	400	K
569				L	0.1	m
570				sigma	5.67E-08	W/m^2.K^4
571						

2. Perform the calculations:

	E584			f_x	=F_ij_Coaxial_parallel_discs($E$569/$E$566,$E$566/$E$569)		
	A	B	C		D	E	F
571							
572			Calculations:				
573					A1 = PI() * R^2 =	0.007854	m^2
574					A2 = A1	0.007854	m^2
575					A3 = 2 * PI() * R * L =	0.031415927	m^2
576							
577			For surface 1:		F_11 = 0, since surface 1 is flat		
578					F_11 + F_12 + F_13 = 1 …. By Summation rule		
579			Therefore:				
580					F_12 + F_13 = 1		
581							
582					Calculate F_12 using the VBA Function:		
583							
584					F_12	0.171572875	
585							
586			Then:		F_13 = (1 - F_12) =	0.828427125	

Note that F_12 is calculated using the VBA Function written above for coaxial, parallel disks. See the Formula bar in the above screen shot.

Continuing, we have:

	A	B	C	D	E	F
587						
588			Calculate F_31 using the reciprocity relation for surfaces 1 and 3:			
589						
590				i.e. A1 * F13 = A3 * F31		
591						
592			Therefore:	F_31 = (A1 * F_13) / A3 =	0.207106781	
593						
594			For surface 3:			
595				F_31 + F_32 + F_33 = 1 …. By Summation rule		
596						
597			Also, for surface 3:	F_31 = F_32 … by Symmetry		
598			Therefore:	F_32 =	0.207106781	
599						
600			Therefore:	F_33 = 1 - (F_31 + F_32)		
601						
602			So, we get:	F_33 =	0.5858	…Ans.
603						
604			Net heat transfer Q_12:			
605				Q_12 = A1 * F_12 * sigma * (T1^4 - T2^4)		
606						
607				Q_12 =	29.340	W … Ans.

Thus:

View factor of the top surface to bottom surface (i.e. F_12) = 0.1716 ... Ans.

View factor of the curved surface to itself (i.e. F_33) = 0.5858 ... Ans.

Net heat transfer between surfaces 1 and 2 = Q_12 = 29.34 W ... Ans.

To plot the variation of F_33 and Q_12 as L varies from 0.1 m to 0.5 m:

3. Set up a Table as shown below. Remember to enter L with relative reference in the equations for F_12, F_13 … etc.

| D613	fx	=F_ij_Coaxial_parallel_discs(C613/$E$566,$E$566/C613)				
	A	B	C	D	E	F	G	H

	L (m)	F_12	F_13	F_31	F_33	Q_12 (W)
613	0.1	0.1716	0.8284	0.2071	0.5858	29.340
614	0.12					
615	0.14					
616	0.16					
617	0.18					
618	0.2					
619	0.22					
620	0.24					
621	0.26					
622	0.28					
623	0.3					
624	0.32					
625	0.34					
626	0.36					
627	0.38					
628	0.4					
629	0.42					
630	0.44					
631	0.46					
632	0.48					
633	0.5					

Equation entered for F_12 in cell D613 can be seen in the Formula bar in the above screen shot.

4. Now, select cells D613 to H613 and drag-copy to the end of the Table, i.e. up to cell H633. Then, immediately, the calculations are done and the Table gets filled up:

L (m)	F_12	F_13	F_31	F_33	Q_12 (W)
0.1	0.1716	0.8284	0.2071	0.5858	29.340
0.12	0.1311	0.8689	0.1810	0.6380	22.415
0.14	0.1027	0.8973	0.1602	0.6795	17.562
0.16	0.0823	0.9177	0.1434	0.7132	14.065
0.18	0.0671	0.9329	0.1296	0.7409	11.482
0.2	0.0557	0.9443	0.1180	0.7639	9.530
0.22	0.0469	0.9531	0.1083	0.7834	8.023
0.24	0.0400	0.9600	0.1000	0.8000	6.840
0.26	0.0345	0.9655	0.0928	0.8143	5.896
0.28	0.0300	0.9700	0.0866	0.8268	5.131
0.3	0.0263	0.9737	0.0811	0.8377	4.503
0.32	0.0233	0.9767	0.0763	0.8474	3.983
0.34	0.0207	0.9793	0.0720	0.8560	3.546
0.36	0.0186	0.9814	0.0682	0.8637	3.177
0.38	0.0167	0.9833	0.0647	0.8706	2.862
0.4	0.0152	0.9848	0.0616	0.8769	2.592
0.42	0.0138	0.9862	0.0587	0.8826	2.357
0.44	0.0126	0.9874	0.0561	0.8878	2.153
0.46	0.0115	0.9885	0.0537	0.8926	1.974
0.48	0.0106	0.9894	0.0515	0.8969	1.816
0.5	0.0098	0.9902	0.0495	0.9010	1.677

5. Now, plot the results in EXCEL:

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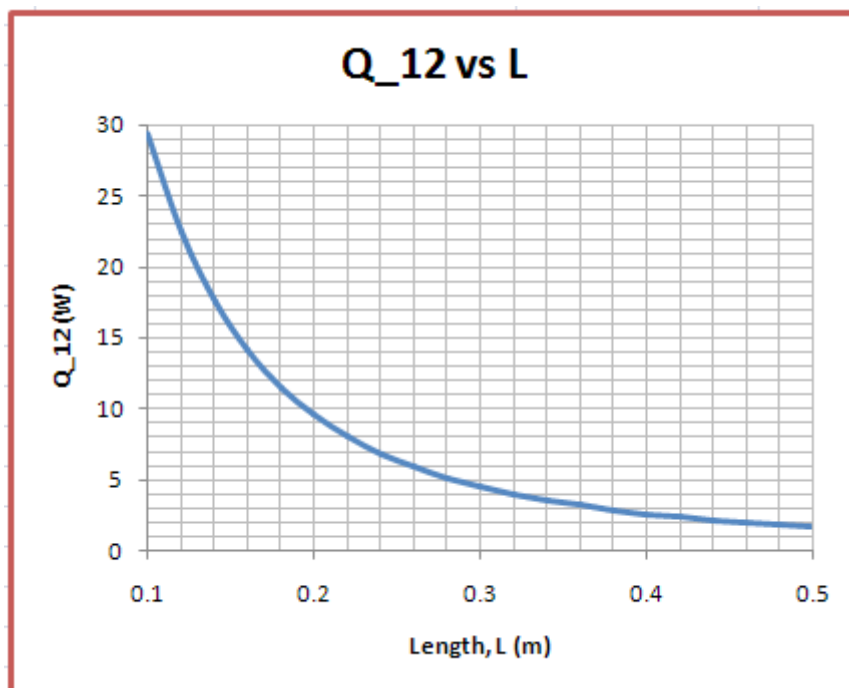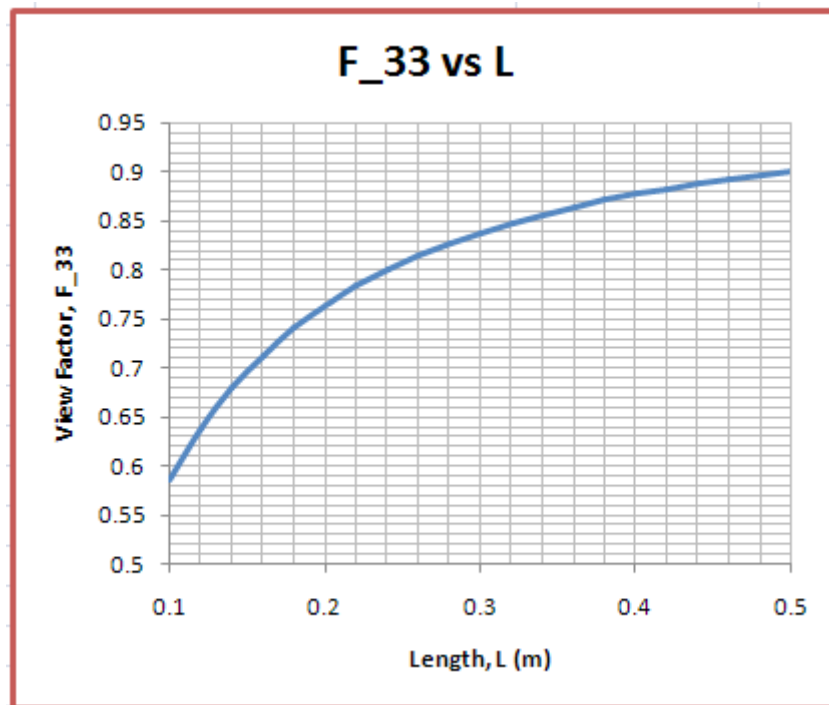
To see the Part II download:

Software Solutions to problems on heat transfer – Radiation Heat Transfer II